# Philips Technical Review

DEALING WITH TECHNICAL PROBLEMS
RELATING TO THE PRODUCTS, PROCESSES AND INVESTIGATIONS OF
THE PHILIPS INDUSTRIES

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#### PROJECTION-TELEVISION RECEIVER

I. THE OPTICAL SYSTEM FOR THE PROJECTION

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A series of articles will be published dealing with various parts of a projection-television receiver for home use. The first of these articles opens with a brief introduction on television in general and then proceeds to deal with the optical system for the projection. For this purpose Philips employ a somewhat modified mirror system with a Schmidt correction plate. The modification consists mainly in the addition of a plane mirror placed obliquely in the path of the light between the spherical mirror and the correction plate. The very fast optical system (numerical aperture 0.62 with a magnifying factor 8.7) is free of third order aberrations with the exception of the curvature of field. The latter has been corrected by giving the screen of the cathode-ray tube a certain curvature. In this manner a perfectly clear picture is obtained on a flat projection screen. This picture is just as bright as a cinema picture and is of such a size (32 cm  $\times$  40 cm) and brightness that the audience can easily observe it in a room with normal or slightly lowered lighting. Due to the compact construction of the optical part and of other parts to be described later, the whole apparatus can be housed in a cabinet of very reasonable dimensions.

In 1939 the technique of television had reached a stage of development where several transmitters were already regularly broadcasting a television programme and receiving sets were on the market. After the war this branch of broadcasting was revived and the number of transmitting stations now regularly working is gradually increasing; at the moment there are several scores of these, most of which are in the U.S.A. (in Europe there is one in London and another in Paris). In addition there are several transmitters of a more or less experimental character (among others there is one at Eindhoven in Holland). With this increasing number of transmitters the demand for television receivers is bound to increase, and it is in this connection that a series of articles are being published in this journal dealing with some important points in the development of these receivers. Before starting on the first of these articles it may well be worth while to sketch briefly the principle of present-day television 1).

The "eye" of the transmitter is the iconoscope 2) or its special form known as the "orthicon", upon the "retina" of which a picture is cast of the scene to be transmitted. This "retina" is periodically scanned by a beam of electrons, thereby producing electrical impulses the amplitude of which corresponds to the brightness of the successively scanned points in the image. These impulses are called video signals and modulate the transmitter.

In the receiver a cathode-ray tube is employed as the light source. Just as in the case of an oscillograph, a beam of electrons produces a spot of light on the luminescent screen of the cathode-ray tube. The beam periodically scans the image plane in synchronism with the beam in the transmitter, the strength of the current in the beam varying according to the modulation of the video signal received.

See for instance J. van der Mark, An experimental television transmitter and receiver, Philips Techn. Rev. 1, 16-21, 1936.

<sup>2)</sup> See the article quoted in footnote 1), particularly page 18.

As a result the points scanned in succession on the screen show variation in brightness corresponding to those of the image transmitted.

To maintain the synchronization between the electron beam in the transmitter and that in the receiver special synchronizing signals are transmitted in addition to the actual video signal.

The television receivers placed on the market from 1936 to 1940 were mostly built for "direct view", the image observed being that produced on the luminescent screen of the tube 3). In particular the large types of tubes for direct view, with a screen diameter larger than 30 cm, are expensive and difficult to handle. To give these large tubes the necessary strength (on a tube face 39 cm in diameter the atmosphere exercises a force much greater than 1000 kg) they have either to be made very thick or given a fairly large curvature. This curvature distorts the image observed, so that the effective area of the face is not proportional to the dimensions of the tube. The danger of implosion makes it necessary to provide special safety measures. Owing to the length of the tube - it increases roughly in proportion to the screen diameter the cabinet has to be made of such large dimensions as to be incompatible with an aesthetic appearance.

All these objections, which arise when a good-sized image is required, were recognized at an early date and are now avoided, or at least considerably reduced, by the projection method. By this method a small image produced on the face of a likewise small cathode-ray tube is projected onto a viewing screen by optical means. Obviously this is the only way if a televised picture is to be viewed by a large audience, for instance in a theatre. But also for the home - and it is exclusively home receivers that we shall be speaking about in this series of articles -- it is very convenient to have a picture of such a size that a fair number of spectators can easily see it without being strictly confined to one particular place. Philips marketed a tube for projection reception as far back as 1937 4). Since then, however, this projection system has undergone considerable development.

Below we shall describe the optical system with which the image of about  $3.6~\mathrm{cm} \times 4.6~\mathrm{cm}$  that is formed on the tube face is projected onto a viewing screen as a bright, sharp and flat picture of

employed in some cabinet models.

4) M. Wolf, The enlarged projection of television pictures, Philips Techn. Rev. 2, 249-253, 1937.

 $32~\mathrm{cm} \times 40~\mathrm{cm}$ . Due to the small dimensions of the tube and the special construction of the optical system it has been possible to house the whole apparatus in a cabinet of moderate dimensions (see fig. l, in the subscript to which various particulars are given).



Fig. 1. Apparatus type SG 860 A for projection-television and for ordinary broadcast reception. The three control knobs in front of the television screen, taken from left to right, are for controlling the sharpness and the brigtness of the picture and adjusting the contrast between the light and dark parts. The five controls underneath the station dial serve for adjusting the selectivity and the volume, changing over from ordinary broadcasting to gramophone music or television, for tuning and for selecting the wave range. Behind the cloth screen below these controls is the loudspeaker. Small extension loudspeakers on either side of the projection screen serve only for television reception. The screen can be left folded down when not tuning in to television broadcasts.

#### Mirrors or lenses?

In order to project the image produced on the cathode-ray tube face onto a viewing screen one may use either concave spherical mirrors, or lenses, or a combination of both these elements. The question is which is to be preferred.

Let us first deal with some advantages of mirrors in general. In the first place they do not show any chromatic aberration. Further, the spherical aberration of a mirror is less than that of a lens with the same diameter and focal length, so that for a given

<sup>3)</sup> See e.g. G. Heller, Television receivers, Philips Techn. Rev. 4, 342-350, 1939, in particular fig. 13. By "direct view" is to be understood here also the observation of the picture via a plane slanting mirror, with the cathode-ray tube having its axis in a vertical position an arrangement employed in some cabinet models.

permissible aberration mirrors can be used with a larger aperture number (i.e. greater speed) than lenses. Moreover, mirrors of high aperture number are comparatively easy to make. Whereas the diameter of a lens is limited to about 50 cm, mirrors of 1.5 metre diameter are quite common, as for instance for searchlights. The cost of a mirror compares favourably with that of a lens of equal performance; a mirror has only one ground and polished face and need not be made of so-called optical glass. This is why astronomers usually prefer mirrors; the Mount Palomar Observatory has a mirror of 5 meters diameter, which surpasses any lens ever made.

But, compared with lenses, mirrors are not entirely free of drawbacks. Owing to the fact that they reflect the light, the image of the object is formed in the path of the incoming light rays, which are thus more or less intercepted. This is the reason why mirrors have found little favour in photography and in the projection of films, where preference is given to fast lenses. This was a disadvantage that weighed rather heavily at first also in the designing of an optical system for the projection of televized images, but by a special arrangement of the components, to which we shall refer presently, this drawback has in so far been overcome as to be more than outweighed by the advantages of a mirror.

#### The mirror system with Schmidt correction plate

We must first recall the fact that a spherical mirror, just like a lens, is subject to a number of image defects, or aberrations 5). Confining our remarks to third-order aberrations, these are to be distinguished as spherical aberration, coma, astigmatism, distortion and curvature of field. In the article quoted in footnote 5) it is shown how a diaphragm placed in the centre of curvature of the mirror neutralizes a number of these defects, leaving only spherical aberration and curvature of field. Further, it is stated that of these two remaining defects the spherical aberration can be eliminated with the aid of a Schmidt correction plate, which has been further discussed in a separate article 6), where a detailed description is given of an extremely simple and inexpensive method developed by Philips for manufacturing these correction plates. The plates made by this method consist of a layer of gelatin applied to a flat glass plate and given the desired profile.

The only image defect (apart from aberrations of a higher order which are of less importance) is the curvature of field: a plane object is sharply focused on a curved surface that is approximately a spherical one. Conversely an object curved in a certain way will be sharply focused on a plane surface. If, therefore, the face of the cathode-ray tube is given a certain curvature then the projection will be flat, so that a sharp image can be obtained on a flat viewing screen. For more details the reader is referred to a later article specially devoted to the cathode-ray tube.

#### Modifications made in the Schmidt system

Schmidt developed his system for photographing the stars. Here the photographic film or plate - and not the object that is to be imaged forms an unavoidable obstruction for some of the light rays that would otherwise be utilized by the optical system. In television projection it is just the other way round. Here we are faced with the question how to apply the cathode-ray tube in such a way that its face, which is here the luminescent object, comes to lie in the right position between the mirror and the correction plate, while the minimum of light is interrupted or lost in any other way. The positioning of the tube is rendered still more difficult owing to the fact that its anode carries a high tension with respect to earth (25 kV) and the tube itself is surrounded by coils for focusing and deflecting the electron beam.

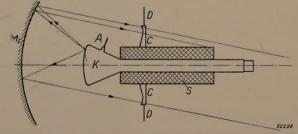


Fig. 2. Simple optical system for television projection but with considerable loss of light through interception. K = cathoderay tube with anode connection A and focusing and deflection ray time with another connection T and steaming and electron coils S.  $M_1 = \text{spherical mirror}$ , C = Schmidt correction plate, D = diaphragm. (In figs 2-5 the outgoing rays are drawn parallel for the sake of simplicity, but actually they converge upon a point on the projection screen, which is a relatively large distance away.)

An obvious construction is illustrated in fig. 2, where the neck of the tube, together with the coils, is passes through an aperture in the correction plate. Even if the tube were short enough to allow of its being accommodated entirely between the mirror and

H. Rinia and P. M. van Alphen, The manufacture of correction plates for Schmidt lenses, Philips Techn. Rev. 9, 349-356. 1947 (No. 12).

<sup>5)</sup> See e.g. W. de Groot, Optical aberrations in lens and mirror systems, Philips Techn. Rev. 9, 301-308, 1947 (No. 10).

the correction plate, the middle part of the correction plate would still lie in the shadow of the tube, so that the fact that in fig. 2 this part has had to be removed for the neck of the tube to pass through it does not constitute any additional drawback. There is, however, the drawback that the tube intercepts some rays coming from the edge of the image and striking the tube or the coils laterally (see fig. 2), though this effect can be reduced by providing for a smaller angle between these rays and the axis, thus by selecting a larger focal distance. This, however, would lead, with a given aperture number (a conception to be dealt with presently), to a larger diameter of the mirror, and this in turn would necessitate a larger cabinet. This solution could only be applied where the space is not limited, as for instance for television projection in a theatre.

A constructional objection against the central aperture in the correction plate is the fact that this aperture makes it extremely difficult to centre the plate. A simple solution for the centering of a plate not having a aperture in the middle will be referred to later on.

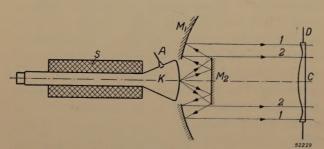


Fig. 3. Modified arrangement of the optical system where the light rays reach the spherical mirror via the plane mirror  $M_2$ . In this model the diameter of the plane mirror is such that the rays I emerging from the centre of the tube face and reflected from the edge of  $M_2$  just pass through the edge of the correction plate C.

A method absolutely avoiding lateral shadow effect of the tube is represented in fig. 3. Here the tube screen is situated in an opening in the centre of the spherical mirror. The rays coming from the tube face strike a plane mirror  $(M_2)$  which reflects them onto the spherical mirror. Thus the dimensions of the tube and the coils are no longer restricted, since the tube and coils do not lie in the path of the light. But now we are faced with another difficulty, viz. the question as to how large the plane mirror has to be. Let us first consider only the rays emitted from the centre of the tube face. In fig. 3 the plane mirror is of such a size that the rays 1 striking its edge just pass through te edge of the correction plate. When we follow the rays coming from the edge of the tube screen and reflected by

the plane mirror (beam between 3 and 5 in fig. 4) we see that these fall partly outside the correction plate and are thus held back by the diaphragm (e.g. 4 and 5 in fig. 4). Thus the useful beam emitted by a point on the edge is narrower than the beam

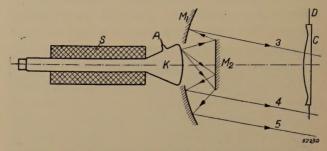


Fig. 4. The same arrangement as in fig. 3 but showing some rays emerging from the edge of the image on the face of the cathode-ray tube. Rays like 4 and 5 fall outside the correction plate.

coming from the centre. This manifests itself in the image on the projection screen as a sort of vignetting, that is to say the intensity of light in the corners is less than in the middle. This phenomenon can be counteracted by making the plane mirror larger, but then still more light rays are intercepted (i.e. the rays 2 in fig. 3). Thus a really satisfactory compromise is not to be found.

Finally it is to be remarked that a system such as that of fig. 3 has the disadvantage of being greater in length than that of fig. 2.

In the Philips optical system for televisoin projection (fig. 5) a plane mirror is also employed but placed at an angle of 45° to the axis of the spherical mirror. This plane mirror is not, optically speaking, situated between the tube screen and the spherical mirror, as is the case in fig. 3, but between the spherical mirror and the correction plate. The screen of the cathode-ray tube protrudes through an opening in the flat mirror, in such a way that the loss of rays coming from the centre of the tube screen is of practically the same magnitude as the loss of rays

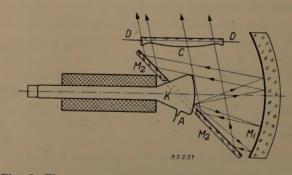


Fig. 5. The arrangement adopted for the Philips optical system. The plane mirror  $M_2$  lies in the light path between the spherical mirror and the correction plate and makes an angle of  $45^{\circ}$  with the axis of the cathode-ray tube.

coming from the edges of the face. Vignetting thus arises to a much less extent than in the case of fig. 3; the tube does not hold back any light laterally (cf. fig. 2) on account of its being behind the mirror, where there is space enough for the coils, fixtures, connecting leads, etc.

The path of the light is folded, as it were, so that the construction of the apparatus can be very compact (fig. 6). The cathode-ray tube and the whole of the optical system can be housed in a dust-proof box, so that the tube face, the two mirrors and the gelatin side of the correction plate are kept clean. Only the flat, glass outer face of the correction plate can accumulate dust, but it is very easy to clean and there is no risk of the system being disturbed. The place occupied by the optical unit in the receiving set is shown in fig. 7.

A second plane mirror is affixed on the inside of the slanting cabinet lid (fig. 1 and 7) to throw the light beam passing through the correction plate onto the projection screen.

#### Characteristic quantities of a Schmidt system

We shall now consider for a moment what quantities are typical of a Schmidt optical system, for we have to refer to these in the next section dealing with the dimensioning of the optical system.

In the first place there is the focal distance, from which follow the distances for the object and image required for a certain magnification. The magnification, too, is a characteristic quantity.

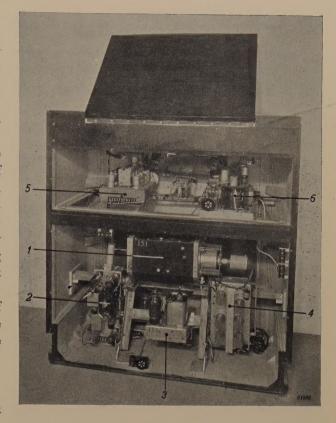


Fig. 7. Inside view of the receiving set SG 860 A (cf. fig. 1) seen from the back.

- I is the optical unit (cf. fig. 6),
- 2 apparatus for deflection of the electron beam,
- 3 high-tension unit (25 kV),
- 4 television receiver (picture and sound),
- 5 receiver for ordinary broadcasting
- 6 rectifier.

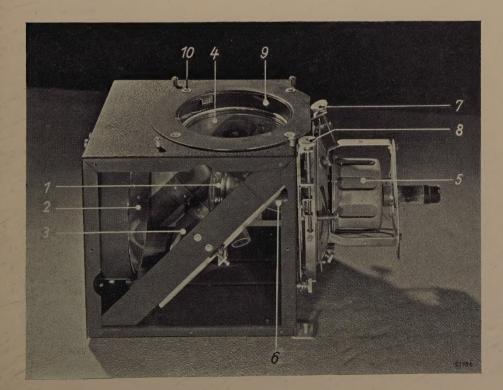


Fig. 6. The Philips optical system for projecting television pictures (one side of the mounting box removed). I is the face of the cathode-ray tube, 2 the spherical mirror, 3 the plane mirror, 4 the correction plate, 5 the focusing coil, 6 the deflection coils, 7 one of the adjusting screws for aligning the axis of the cathode-ray tube, 8 the screw for adjusting the distance between the tube screen and the spherical mirror, 9 screws fixing the correction plate holder after the plate has been set to the correct height, 10 fixing screws around which some play is left in the rim of the correction plate holder for the purpose of centering.

Just as a parabolic mirror gives a good image only with one object distance or image distance, viz. infinite, so a particular Schmidt system is suitable for only one such distance, thus for one particular magnification 7). If the system were to be used

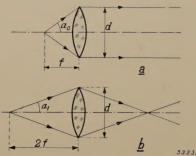


Fig. 8. a) Simple lens with light source in the focus and infinite projection of the image. The relative is understood to be  $d/f = 2 \tan a_0$ .

b) The same lens with true-to-size projection. Here the relative aperture is d/2f=2 tan  $a_1$ .

with some other magnification then the spherical aberration would not be completely eliminated. For every correction plate it is therefore necessary to specify not only the radius of curvature of the corresponding mirror but also the magnification for which it is calculated, the more so since this magnification cannot easily be derived from the form of a particular specimen.

Further, the "speed" is an important quantity in an optical system. In the case of lenses this is usually expressed by the relative aperture. For a simple lens this represents the ratio of the diameter d of the lens (or of the diaphragm) to the focal distance f. If the apex angle of the cone of rays striking the lens from a light point placed on the axis is 2a and if the light point lies in the focus (where  $\alpha$  has the value  $\alpha_0$ ; fig. 8a), so that the image is formed at an infinite distance then the aperture number  $d/f = 2 \tan a_0$ , If the path of the light rays is reversed then we get the situation which arises in photography when the object distance can be regarded as infinite, as is often the case. When, however, an object is photographed from close by, e.g. so that the picture is the actual size of the object (fig. 8b), then the speed of the lens is determined by an angle  $a_1 < a_0$  (in this case  $\tan a_1 = \frac{1}{2} \tan a_0$ ). Accordingly, when taking a close-up photograph a longer exposure time is required than when the focusing is infinite (under otherwise the same conditions).

From this it appears that when specifying the value of d/f one should really mention the object or image distance used (or, what comes to the same

thing, the magnification). Moreover, it is in fact not tan a, that might be derived from these data, but rather sina that gives a direct measure for the speed of an optical system. In relatively low-speed optical systems, such as those of the usual cameras, the value of a is so small that the difference between tana and sina is of no significance, but with the very high speeds that can be reached with the Schmidt system this difference is indeed of importance. The quantity sina, also called numerical aperture, has long been commonly used for microscope objectives. It seems to us desirable to introduce this quantity also for Schmidt optical system, the more so since  $\sin^2 \alpha$  is the light-gathering power of the optical system for an object emitting rays according to Lambert's cosine law. (By "light-gathering power" is understood the ratio that the quantity of light entering bears to the total light emitted by the object.) The angle  $\alpha$  is then to be understood as being half the apex angle of the cone formed by the rays which come from the centre of the image screen and farther on just pass through the edge of the correction plate (see fig. 9, where for the sake of simplicity the Schmidt system is represented in the simple form of fig. 2).

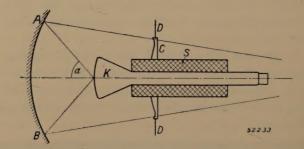


Fig. 9. The numerical aperture of a Schmidt system is understood as being  $\sin a$ , where in the case of an optical system for television projection a is half the apex angle of the widest cone of rays emerging from the centre of the tube face and after reflection from the spherical mirror passing through the edge of the correction plate.

Of course it is thereby assumed that the spherical mirror is large enough to reflect these rays, its diameter from edge to edge being at least AB (fig. 9). However, a still larger mirror does not at all imply a larger numerical aperture. It is therefore incorrect to take the diameter of this mirror as a measure for the speed, as is sometimes done. This does not alter the fact that it does serve a purpose to take a mirror larger than AB, for this improves the situation for rays coming from the edge of the image on the face of the cathode-ray tube and reduces vignetting.

Finally there has to be mentioned as a characteristic quantity of an optical system the loss factor, a factor indicating what fraction of the amount of light entering the optical system is lost through interception, imperfect reflection and absorption.

<sup>7)</sup> Apart from the reciprocal value corresponding to the inverse direction of the light rays.

#### Dimensioning of the optical system

All dimensions of the optical system bear, roughly speaking, a direct relation to the focal distance. When aiming at keeping the size of the cabinet within reasonable limits one would therefore be inclined to choose a small focal distance. However, the cathode-ray tube sets a limit to this. This tube cannot be reduced to any size at will, because account has to be taken, inter alia, of the question whether a sufficiently fine light spot can be obtained. One might, it is true, try to work with a small magnification, but this would be disadvantageous because then the tube, being comperatively large, would intercept too much light; it may be taken as a general rule that the diameter of the tube face should be at most no more than half that of the correction plate, so that no more than one quarter of the light is intercepted.

The following rough calculation is intended to show the relation between the desired size and brightness of the projected image on the one hand and the diameter and the luminous flux of the cathode-ray tube together with some data of the optical system on the other hand.

We shall start from the size of the projected image  $32 \text{ cm} \times 40 \text{ cm}$ , area  $0.128 \text{ m}^2$ ) and its brightness, assuming, to start with, that the latter is required to be equal to that of a good screen picture in a cinema (about 32 candles/m<sup>2</sup>) 8). If the light emitted from the projection screen were absolutely diffuse then in order to reach this required brightness a luminous flux of  $0.128 \times 32 \times \pi = 13$  lumens would have to fall on the screen. Much less is required, however, if the screen is made of a translucent material, such as frosted glass. The fact is that when glass is frosted in a certain way the light falling perpendicularly on the back of the screen mainly emerges at the front also perpendicularly and is scattered only to a very small extent (we shall revert to this presently). Such a selectively scattering screen gives a gain, for instance, of a factor well over 4, so that a luminous flux of only 3 lumens is then required to fall on the screen. There is, however, some loss of light in the optical system through which the luminous flux has to pass, firstly on account of the interception previously referred to (about 25%) and further in the reflection from the mirrors and the passage of the light through the correction plate (together likewise about 25%), so that the loss factor amounts to about 0.5. Therefore, in order to get a yield of 3

8) In American literature brightness is usually expressed in foot-lamberts. The relation between this unit and the candle/m² unit is: 1 ft-lambert = 3.43 candles/m².

lumens from the optical system about 6 lumens has to enter it.

But the luminous flux coming from the cathode-ray tube has to be even larger. The luminous flux taken up by the optical system expressed as a fraction of the luminous flux emitted by the cathode-ray tube is equal to the square of the numerical aperture, as already stated. If this aperture is say 0.62 then  $6/0.62^2 \approx 15$  lumens has to be emitted from the tube face. It has been assumed again that the emission from the tube face follows approximately Lambert's cosine law; therefore the luminous intensity in the axial direction must be  $15/\pi \approx 5$  candles. The light yield of luminescent substances lies between 1.6 and 3 candles/watt (averaging 2 candles/watt) so that for 5 candles luminous intensity about 2.5 W is required in the electron beam. The question as to what voltage and current are required to reach this wattage will be answered in a later article, but we may already say that the cathode-ray tube has sufficient reserve capacity to allow of a short peak current intensity 5 times as great as that mentioned. Thus a brightness can be reached 3.5 to 4 times as great as that from which we started (32 candles/m<sup>2</sup>). (The fact that the brightness increases less than proportionately with the current intensity is due mainly to saturation phenomena in the luminescent substances.)

Regarding the cathode-ray tube we may also state that it has been found possible to reduce the face diameter to 6.3 cm (the size of the image on the face is about 3.6 cm  $\times$  4.6 cm). According to the rule given above the correction plate should have a diameter about twice that of the tube face, thus in this case about 12 cm. Taking this diameter and the figure of 0.62 given in the foregoing for the numerical aperture, it follows that a focal distance of about 10 cm is required. With these data given, all other dimensions of the optical system are determined.

The quality that can be reached with the Philips optical system is such that more than twice the present number of lines per image could easily be dealt with. (However, difficulties of quite a different nature, into which we cannot enter here, prevent such a raising of the number of lines.)

Fig. 10 is a photograph of an image televized by the experimental transmitter at Eindhoven and received with an apparatus of the type shown in fig. 1.

#### Focusing the optical system

To get a proper projection with a fast optical

system it is necessary to focus very carefully. In the first place the centre of the correction plate must exactly coincide with the centre of curvature of the spherical mirror — or, in the case of the Philips optical system, with the projection of this centre on the plane mirror . This can be brought about in two stages: 1) by moving the correction plate in the axial direction until the centre of curvature (or its projection) just lies on the face

the correction plate slightly upward or downward the position can be found where no parallax is to be observed between the mark on the correction plate and its projection on the mirrors. The correction plate holder is then secured in this position by means of the screws of which one is indicated by the number 9 in fig. 6. The second adjustment, shifting the correction plate in its own plane, has been made possible by leaving some play in the



Fig. 10. Photograph of a television picture transmitted from the experimental station at Eindhoven and received with an apparatus of the type illustrated in fig. 1. Number of lines 567 (interlaced scanning 9), 50 frames per second.

of the plate; 2) by shifting the correction plate perpendicularly to its axis until its centre coincides with the point just referred to.

The importance of this latter centering will be clear when it is borne in mind that the thickness of the correction plate at the edge varies rapidly with the distance from the centre (see the article referred to in footnote <sup>6</sup>)), so that only a very small deviation form the correct centering is sufficient to spoil the whole correction.

To facilitate these adjustments the centre of the correction plate is indicated by the point of a V-shaped mark. The two mirrors form a true image of this mark which just falls on the correction plate when this is at the correct height. By screwing

holes in the rim of the correction plate holder through which the fixing screws are passed (10 in fig. 6). The correction plate is moved about until the point of the V mark coincides with the point of its projection on the mirrors, so that two V's together form a cross, after which the plate is secured by tightening the screws marked 10.

In the second place the spherical face of the cathode-ray tube has to be correctly positioned with respect to the optical system. In the manufacture of the tube care has already been taken to give the face the radius of curvature corresponding to the optical system. The tube holder has been fitted into the receiver in such a way that its distance from the spherical mirror can be adjusted (see fig. 6). Furthermore, this tube holder allows of some play in the angle between the axis of the tube and that of the mirror, so that it is possible to align the centre

<sup>9)</sup> For interlaced scanning see e.g. Philips Techn. Rev. 3, 285-291, 1938.

of curvature of the tube face in the axis of the mirror.

#### The projection screen

As already remarked in the introduction, one of the main advantages of a large picture is that several persons in one room can follow the television broadcast at their ease. To obtain a picture of such dimensions the projection method must be applied. We have also seen that by using a selectively scattering screen we have a welcome gain in the brightness of the picture, but it is to be noted that this can only be obtained at the cost of a reduction in the number of the audience or reduced freedom in the choice of position from which the picture can be viewed. Consequently we have to find a compromise.

We have already mentioned that frosted glass is a suitable material for a screen with selective scattering. Such a material can be characterized by its scattering curve  $N = f(\Theta)$  (fig. 11b; for the meaning of N and  $\Theta$  see fig. 11a and its subscript). In principle only two quantities of such a scattering curve are of importance: the maximum scattering factor  $N_0$  and the angle at which N is a certain fraction of  $N_0$ . For this fraction we chose 1/2; and we therefore speak of the half-value angle  $\Theta_{1/2}$ . This may be regarded as half the apex angle of a scattering cone forming approximately the boundary of the space within which a good view is possible. The scattering curves of different kinds of frosted glass answer approximately the equation  $N_0 \sin^2 \Theta_{1/2} = {
m constant}, {
m in which expression is}$ again given to the compromise that has to be made between  $N_0$  and  $\Theta_{1/2}$ .

An objection attaching to a material like frosted glass is that in vertical planes there is just as much scattering as in the horizontal direction. Now in vertical planes a much smaller half-value angle suffices, because the eye levels of tall and short, standing and sitting viewers differ much less than the width of the space in which several people have to be placed. A gain in brightness could therefore be reached by using a projection screen ribbed in such a manner that the half-value angle is smaller in the vertical plane than in the horizontal plane. The ribs, however, would have to be very fine, in fact smaller than the projection of the light spot on the projection screen. With the  $8.7 \times$  enlargement applied, the diameter of the image of the light spot is about 0.6 mm. Such a screen is rather difficult to make. Moreover, owing to the regular structure so-called moiré effects might easily arise.

So far Philips have been using a frosted glass projection screen with  $N_0=$  about 4 and  $\Theta_{1/a}=$  about  $17^{\circ}$  (fig. 11b). This angle is large enough for the picture to be easily observed by a fair number

of spectators. Just as in a cinema, the distance between the audience and the screen must be at least 5 times the picture height; thus 1.6 m for a picture of 32 cm × 40 cm. This is necessary to be able to perceive the whole picture at one glance. At this distance the edges of the picture are observed at an angle of about 5° from the perpendicular to the centre of the picture, that is to say the image of the edges is cast upon the retina at a place where the visual acuity is reduced to 1/3 of that in the middle of the fovea (the central part of the yellow spot). which experience shows to be just admissible. With  $\Theta_{\nu_{o}} = 17^{\circ}$  at the minimum distance of 1.6 m there is room for two people. There are, moreover, other reasons why the minimum distance has to be about 1.6 meters: at a shorter distance one can see the lines from which the picture is built op.

Finally, we have something to say about the lighting of the room in which the receiver is placed

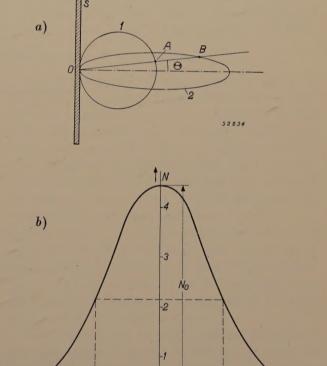


Fig. 11. a) When S represents a white, diffuse-reflecting plate (e.g. MgO) then the light incident at right-angles from the right is scattered in such a way that the part OA cut by the circle I on a radius vector (making an angle  $\Theta$  with the perperdicular to S) is a measure for the intensity which S possesses in the direction given by the angle  $\Theta$ . If, on the other hand,  $\Theta$  is a frosted glass plate illuminated with equal intensity but this time from the left, then instead of the circle we find e.g. the ellipse 2. The ratio OB/OA is called N.

10°

b) Scattering curve  $N=\mathbf{f}\left(\Theta\right)$  of a particular kind of frosted

and about the framing of the projection screen. In a blacked-out room the television picture is certainly very bright, but is gives an unnaturel impression and it is difficult to get that feeling of "living" with the scene. In the other extreme case of a very brilliantly lighted room attention is easily distracted from the picture, because it is less bright than its surroundings. The most favourable conditions for viewing are obtained when soft daylight is allowed to enter the room or normal or slightly toned down artificial lighting is used.

Another important point is the framing of the picture. It is best to use light colours, the effect being comparable to that of the frame round a painting or the white mounting round an etching; it appears to be an invaluable help for the imagination of the observers, bringing much more life into the picture. This effect is considerably enhanced by the small auxiliary loudspeakers set up on either side of the screen and reproducing mainly that part of the sound spectrum that is of most importance for speech.

## THE RATIONALIZED GIORGI SYSTEM WITH ABSOLUTE VOLT AND AMPERE AS APPLIED IN ELECTRICAL ENGINEERING

by P. CORNELIUS.

537.713

Various systems of electrical units are in use today, these having been introduced in the course of the development of the theory or practical application of electricity in various fields. By an efficient combination of the conventional units the rationalized Giorgi system with absolute volt and ampere has been developed and made suitable to cover the whole range of electromagnetism. In this system several electrical quantities are given not only a different value but also a difinition different from that previously employed. As a result the current in conductors as well as the electrical and the magnetic field can all be dealt with in an analogous manner. In this article, after a brief review of the older systems, the rationalized Giorgi units are summarised and explained. The most important formulae for electromagnetism are tabulated in the form which they assume when expressed in these units. This table and several of the quantities occuring therein are discussed.

#### Introduction

The electrostatic c.g.s. system with its concepts of charge and potential follows naturally when one starts from the Coulomb's law and adopts the mechanical units cm, g, sec with the accompanying concepts of force and work as given.

The electromagnetic c.g.s. system with the concepts of magnet pole, magnetic field strength and electric current is obtained just as naturally when one starts from Coulomb's magnetic law, and the relation between current and magnetism is described for instance with the aid of the Biot and Savart law.

From electrostatics and electromagnetism one can arrive at a combined theory, based on Maxwell's laws with the inherent concepts of field. If, then, the two c.g.s. systems are taken as being equivalent, the so-called Gauss system is an obvious solution.

The theory of electromagnetism as represented by the Gauss system and the requirements of electrical engineering, in which "practical units" are employed, are reconciled in an extremely simple manner by means of the rationalized Giorgi system with absolute volt and ampere. This system is built up from the conceps of current and voltage. The concept "current", represented as a charge in motion or as the cause of magnetic phenomena, plays an important part also in the line of thought covering the three c.g.s. systems. The concept "voltage" on the other hand occupies a less prominent place in these systems and is represented therein as a difference in potential or as the line integral of the electric field strength. In electrical engineering, however, this concept of voltage is so fundamental as to be of predominant importance, for here it is commonly said that "the voltage drives the current through an impedance"

and "that the voltage between two conductors causes an electric field". Regarded from this point of view the concepts of electromagnetism are of course often given a different aspect.

Bearing this in mind, the rationalized Giorgi system 1) may be summarized as follows.

#### The rationalized Giorgi system

The rationalized  $^2$ ) Giorgi system is based, for all electrical and magnetic quantities and thus also for field quantities, upon the volt and ampere, the units with which everyone is familiar. Further, the usual electrical units of the ohm  $(\Omega)$ , the coulomb (C), the farad (F) and the henry (H) are maintained. The electrical field strength E is measured in volts per meter (V/m), and the magnetic field strength H in amperes per meter (A/m).

The electric flux  $\Psi$  passing from positive to negative charges is measured, like the charge Q itself, in ampere-seconds or coulombs. The displacement D (a better name is electric induction) may be regarded as electric flux density and is therefore measured in  $A \cdot \sec/m^2$  or  $C/m^2$ .

The magnetic flux  $\Phi$ , which encircles a conductor through which a current is flowing, or which is induced by the molecular circuit currents of permanent magnets or magnetized iron, is given in

We are discussing here exclusively the rationalized Giorgi system. No attention is paid to the question whether the formulae given here may also hold for the

non-rationalized system.

<sup>1)</sup> Cf. W. de Groot [4]. (Figures between brackets refer to the bibliography at the end of this article). There the historical development of electromagnetism is outlined and the position occupied therein by the Giorgi system is shown. It also deals with the difference between the "absolute" and the "international" volt and ampere. The present article will show mainly how the rationalized Giorgi system with absolute volt and ampere can be applied in electromagnetism.

volt-seconds, also called webers (V·sec = Wb). The magnetic induction B can be regarded as magnetic flux density and is therefore measured in V·sec/m² or Wb/m².

Giorgi chose as the unit of length the metre instead of the centimetre, so as to give a simple relation between electrical and mechanical quantities (see below). The Wb takes the place of the maxwell, the Wb/m² the place of the gauss and the A/m the place of the oersted. In engineering the A/cm, under the name of ampere-turns per cm, has long been adopted in the place of the oersted. It is then a very short step to the A/m. The V/cm and also the Giorgi unit V/m have likewise already become of common use.

In the Gauss system the displacement D has the same dimensions as the electric field strength E at the same point, whilst moreover in vacuum D and E are identical. Giorgi on the other hand ascribes to D a value and a dimension different from E, not only in matter but also in vacuum. The same applies for the magnetic field strength H and the magnetic induction B. The advantages that are to be set against this complication will be seen farther on.

It is to be pointed out that this complication in the Giorgi system is partly met with also in the electrostatic and the electromagnetic c.g.s. system. It is true that in the electrostatic system D=E in vacuum but  $B=H/c^2$ ; in the electromagnetic system on the other hand B=H in vacuum but  $D=E/c^2$ . As opposed to the advantage of the Gauss system that in vacuum D=E and B=H, is the drawback that we find the factor c occurring in many important formulae.

From this it is to be seen that the difference between the electrical systems is not merely a question of numerical factors. Whereas calculating with say feet instead of metres does not in principle present any difficulties, the trouble when changing over from one electrical system to another is that quantities change not only in value but also in definition and dimension. This fact is not always sufficiently realized when the three c.g.s. systems are employed, because in these systems the electrical quantities are expressed in the same mechanical units.

It is, therefore, a great advantage of the rationalized Giorgi system that the definition, or at least the meaning, of the electrical quantities can easily be recognized from the unit.

From the internationally recognized decimal system of mass and length units Giorgi took the kilogram as mass unit and, as mentioned above, the metre as length unit. The unit of force in the Giorgi system is the newton (N), the force inducing in a mass of 1 kg an acceleration of 1 m/sec<sup>2</sup> (in some publications the rather misleading expression m/sec/sec is used instead). Provided one uses the "absolute" (instead of the "international") volt and ampere, the unit of mechanical work, the newton-metre, and the unit of electrical energy,

the watt-second (= volt-ampere-second = joule (J)) are exactly identical:

$$1 \cdot N \cdot m = 1 \cdot V \cdot A \cdot sec.$$

The relation between the voltage, charge and current expressed respectively in the units of the volt, coulomb and ampere and the forces occuring in the corresponding fields thus assumes a simple form.

As an application of the Giorgi system a summary is given of the more important relations and laws of electromagnetism expressed in rationalized Giorgi units (table I). This table is discussed below.

At the end of this article two other tables are given in which the formulae and units of the older systems are compared with those of the rationalized Giorgi system.

We shall now discuss some of the quantities given in table I and explain the line of thought underlying the compilation of this table.

#### Field quantities

Owing to the historical development of electromagnetism, to which expression is given in the c.g.s. systems, when speaking of field quantities one is accustomed to thinking in the first place of forces. The electric field strength E is there defined as the force acting on the unit of charge and the magnetic induction B as the force acting on the unit of current element. According to this point of view D and H are auxiliary quantities for describing fields in matter and are superfluous when describing vacuum fields.

Since in the Giorgi system the field units are derived from the voltage and current units volt and ampere, it is advisable to replace the usual definitions of field quantities by others based upon measurements of voltage and current. When represented in this manner all four field quantities are of equivalent importance. E and B describe the electromagnetic field with the aid of voltage measurements, D and H with the aid of current measurements, regardless whether the field occurs in matter or in vacuum.

It is then found that by employing rationalized Giorgi units the current in conductors, the "current field", as well as the electric and the magnetic field can be dealt with in a manner which in many respects is similar.

#### The current field

In homogeneous as well as inhomogeneous cases<sup>3</sup>)

There may be inhomogeneity in the current distribution, material, etc.

Table I. Some formulae expressed in rationalized Giorgi units

Current field	Electric field	Magnetic field
Ohm's law $I = G V$ (A) (S) (V) $S  (siemens) = A/V$	Law of capacity $(\Psi = Q)$ $\Psi = C V$ $(C)   (F)   (V)$ $C   (coulomb) = A \cdot sec$ $F = A \cdot sec/V$	$Law  ext{ of self-induction} \ (1  ext{ turn}) \ oldsymbol{\Phi} = L  ext{ } I \ (Wb)  ext{ } (H)  ext{ } (A) \ Wb  ext{ (weber)} = V \cdot \sec A \ H = V \cdot \sec A$
	Vector equations	
$S = \gamma E $ $(A/m^2)  (S/m)  (V/m)$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$rac{B}{ ext{(Wb/m}^2)} = rac{\mu}{ ext{(H/m)}} rac{H}{ ext{(A/m)}}$

Relations between the field quantities

		a) in homogeneous fields
I	= S A	$\Psi = D A \qquad   \qquad \Phi = B A$
(A)	$(A/m^2)$ $(m^2)$	(C) $(C/m^2)$ $(m^2)$ $(Wb)$ $(Wb/m^3)$ $(m^2)$
V	= E s	$V = E  s  I = H  s^*)$
(V)	(V/m) (m)	$ (V) \qquad (V/m)  (m) \qquad (A) \qquad (A/m)  (m) $
		b) general
I	$= \iint S_n dA$	$\Psi = \iint D_n  dA \qquad $
(A)	$(\mathrm{A/m^2})$ $(\mathrm{m^2})$	(C) $(\mathring{C}/m^3)$ (m) $(\mathring{W}b/m^2)$ (m <sup>2</sup> )
V	$=\int E_s  \mathrm{d}s$	$V = \int E_s  \mathrm{d}s$ $I = \oint H_s  \mathrm{d}s$
<b>(</b> V)	(V/m) (m)	$(V) \qquad (V/m)  (m) \qquad \qquad (A) \qquad (A/m)  (m)$

Homogeneous cases

Conductance	Capacity	Self-induction (1 turn)
$G = \gamma A/s$	$C = \varepsilon A/s$	$L = \mu A/s$
(S) $(S/m) (m^2/m)$	$(F) \qquad (F/m)  (m^2/m)$	(H) $(H/m) (m^2/m)$
Power	Ene	rgy
Conductor	Capacitor $(\Psi = Q)$	Coil (1 turn)
P = I V	$W = \frac{1}{2} \Psi \cdot V$	$W = \frac{1}{2} \Phi I$
(W) (A) (V)	(W·sec) (C) (V)	$(W \cdot sec)$ $(Wb)$ $(A)$
Space density of the power	Space densi	ty of energy
$(\text{in W/m}^3) = S  E$		$( ext{in W} \cdot  ext{sec/m}^3) =  ext{1 \over 2} B H$
$(A/m^2)$ $(V/m)$	$(C/m^2)$ $(V/m)$	$(Wb/m^2)$ $(A/m)$

Maxwell equations

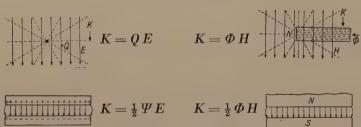
$$\oint E_s \, ds = - \dot{\Phi} \dots (V)$$

$$\oint H_s \, ds = I + \dot{\Psi} \dots (A)$$

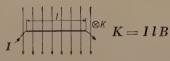
For the surface A enclosing a part of the space:

$$I \equiv \oint \oint S_n \, dA = - \stackrel{\circ}{Q} \dots \quad (A)$$
 $\Psi \equiv \oint \oint D_n \, dA = Q \dots \quad (C)$ 
 $\Phi \equiv \oint \oint B_n \, dA = 0 \dots (Wb)$ 

Laws of force







Energy equivalent: 1 V·A·sec = 1 N·m

Relation to velocity of light:  $\varepsilon_0 \, \mu_0 \, c^2 \, = \, 1$ 

Value of  $\mu_0$ :  $\mu_0 = 4 \pi/10^7 \dots (H/m)$ 

<sup>\*)</sup> In this table I represents the entire current surrounded by the flux, in technical language the ampere-turns; s in the case of a long coil is the length of the coil, and in the case of a toroid the length of the circular centre line.

the current in conductors is expressed, as stated by Ohm, by:

$$I = GV$$

(I...(A); V...(V); G is the conductance, unit  $A/V = \Omega^{-1} = \text{mho} = \text{siemens (S)}.$ 

In a homogeneous case we can divide the current by the cross section perpendicular to its direction and the voltage by the length in the direction of current. We may therefore write Ohm's law for specific quantities and take these quantities in the direction in which they have the greatest value. Thus we arrive at the vector equation of the current field 4):

$$S = \gamma E$$

(S denotes the current density in  $A/m^2$ : E is the electric field strength in V/m and  $\gamma$  the conductivity in  $A/V \cdot m = S/m$ ). This relation, although it has been deduced for a homogeneous case, can likewise serve for describing inhomogeneous cases. In most cases the quantity  $\gamma$  is a scalar material constant. The numerical value of  $\gamma$  is equal to the conductance of the "unit conductor" (with dimensions: length = 1 m; cross section = 1 m²) of the material in question in the case of homogeneous distribution of current.

#### The electric field

When the voltage on a capacitor is increased by 1 V then the same current impulse  $\int I dt \dots$ (A · sec) or quantity of charge Q . . . (coulomb (C) =  $A \cdot sec$ ) is always taken up, regardless of the rate at which the voltage is increased. The value C of a capacitor denotes the magnitude of this current impulse or quantity of charge; the unit is the farad  $(F) = A \cdot \sec/V = C/V$ . The concept of capacitance and the phenomena related thereto may be described, following Faraday and Maxwell, as follows. Between the plates of a charged capacitor is an electric field. This field accumulates the energy . .  $(V \cdot A \cdot sec)$  used for the charge, and the voltage V. (V) between the plates sets up an electric flux  $\Psi \dots$  (A · sec), passing from the positive charge  $Q \dots (A \cdot sec)$  on one plate to the negative charge Q on the other plate, where the general relation is:

$$\Psi = 0 = CV$$
.

In a homogeneous case (plane capacitor in which the distance between the plates is small compared with the plate surface) the electric flux can be divided by the cross section perpendicular to its direction and the voltage by the length in that direction. Thus we write the ordinary law of capacitance for specific quantities and take these specific quantities in the direction in which they have the greatest value <sup>5</sup>).

Thus we arrive at the vector equation of the electric field:

$$\mathbf{D} = \varepsilon \mathbf{E}$$

(D denotes the displacement, i.e. electric flux density in  $C/m^2$ ;  $E \dots (V/m)$ ;  $\varepsilon$  is the "(absolute) dielectric constant" in  $A \cdot \sec/V \cdot m = F/m$ ).

This relation, though deduced for a homogeneous case, can likewise serve for describing inhomogeneous cases. In the simplest case the quantity  $\varepsilon$  is a scalar constant of the dielectric medium (matter or vacuum). The numerical value of  $\varepsilon$  is equal to the capacity of the "unit capacitor" (with dimensions: plate separation = 1 m; plate surface = 1 m²) in the case of homogeneous field distribution 6).  $\varepsilon$ ... (F/m) performs the same function for the electric flux, as the conductivity  $\gamma$ ... (S/m) for the electric current.

When using the absolute volt and ampere and substituting the velocity of light  $c=2.99776\times10^8 \text{m/}$  sec, the value of the dielectric constant of the vacuum, called the "electric induction constant", is:

$$\varepsilon_0 = 10^7/4\pi c^2 \approx 8.855 \times 10^{-12} \dots ({\rm F/m}).$$

<sup>6)</sup> Homogeneous field distribution can be obtained by means of a guard ring, see fig. 1.

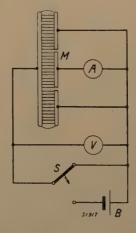


Fig. 1. Plane capacitor with guard ring. Inhomogeneity of the field occurs only at the outer edge of the ring. Only the charge of the middle part M is measured, where the field is homogeneous. A ballistic ammeter; B battery; S switch for charging and discharging the capacitor; V voltmeter.

<sup>4)</sup> Vector quantities, which in contrast with scalar quantities are characterized not only by a dimension and a numerical value but also by direction, are denoted by letters printed in heavy type (except in the tables). The equation given here expresses the fact that S and E bear a certain relation in value and also have the same direction.

<sup>5)</sup> The electric flux through a surface can be measured with the aid of electric induction. See e.g. R. W. Pohl [3], p. 29.

From the Maxwell theory we have the fundamental relation

$$\varepsilon_0 \mu_0 c^2 = 1$$
.

To derive the value of  $\varepsilon_0$  one therefore has to remember, in addition to the velocity of light, only the value of the "magnetic induction constant"  $\mu_0=4\pi/10^7\ldots$  (H/m). The meaning of  $\mu_0$  is explained farther on.

The dielectric constant of a material dielectric is usually denoted by the product of its "relative dielectric constant"  $\varepsilon_r$  and  $\varepsilon_0$ :

$$\varepsilon = \varepsilon_r \varepsilon_0 \dots (F/m).$$

 $\varepsilon_r$  is a non-dimensional number, the material constant known of old.

The force exercised upon a charge  $Q \dots (C)$  in an electric field having the field strength  $E \dots (V/m)$  is:

$$\mathbf{K} = Q\mathbf{E}\dots(\mathbf{N}).$$

Whenever it is desired to regard the electric field strength E as a force vector this can also be measured not in V/m but in the identical unit N/C.

With the aid of the equation given above Coulomb's law can be deduced by calculating the field strength at the point where there is a charge  $Q_2$  from the flux proceeding in spherical symmetry from a charge  $Q_1$ .

#### The magnetic field

When the current flowing through a loss-free coil is increased by 1 A, then the same voltage impulse  $\int V \, dt \dots (V \cdot \sec)$  always arises between the ends of the coil, regardless of the rate at which the current increases. The self-inductance L of a coil indicates the magnitude of this voltage impulse; the unit is the henry  $(H) = V \cdot \sec/A$ .

According to the method of representation followed by Faraday and Maxwell a current is surrounded by a magnetic field. This field accummulates the energy...  $(V \cdot A \cdot \sec)$  that is used for starting a current. The current  $I \dots (A)$  flowing in a coil of a single turn thereby gives rise to a magnetic flux  $\Phi \dots (V \cdot \sec = \text{weber } (Wb))$  enveloping the wire, the general relation applying for a single turn being:

$$\Phi = L I$$
.

Let us now consider a long coil of a single turn consisting, for instance, of a wide strip of metal. The width of the strip is the length of the coil (see fig. 2) 7). In this coil the field is practically homogeneous, and negligible in the space outside the coil. Practically the entire "magnetomotive force".

(A) of the enclosed current is used to drive the magnetic flux through the length of the coil.

In this homogeneous case the magnetic flux can be divided by the cross section perpendicular to its direction and the current by the length in that

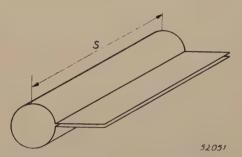


Fig. 2. Single-turn coil made from a wide strip of metal, the width of the strip being the length s of the coil.

direction. We therefore write the normal selfinduction law for specific quantities and take these quantities in the direction in which they have the greatest value <sup>8</sup>). Thus we arrive at the vector equation of the magnetic field:

$$\mathbf{B} = \mu \mathbf{H}$$

(B denotes the magnetic induction, i.e. magnetic flux density, in  $Wb/m^2$ ; H is the magnetic field strength in A/m and  $\mu$  the "(absolute) permeability" in  $V \cdot \sec/A \cdot m = H/m$ ).

Although deduced for a homogeneous case, this relation can also serve for describing inhomogeneous cases. In the simplest case the quantity  $\mu$  is a scalar constant of the magnetic medium (matter or vacuum). The numerical value of  $\mu$  is equal to the self-inductance of the "unit coil" (1 turn) having a length of 1 m and an enclosed area of 1 m²) in the case of homogeneous field distribution 9).  $\mu$  ... (H/m) performs the same function for the magnetic flux, as  $\epsilon$  ... (F/m) does for the electric flux (S/m) for the current.

In the case of ferromagnetic media  $\mu$  is often still a scalar quantity but depending on H and the previous history of the material (hysteresis).

The permeability of the vacuum, called the "magnetic induction constant", when using the

<sup>7)</sup> This is the ideal solenoid, without any leakage of the magnetic field between the windings, which occurs in the case of a solenoid whose windings are mutually insulated.

<sup>8)</sup> The magnetic flux through a surface can be measured by means of an induction test. See e.g. R. W. Pohl [3], p. 74. The "magnetic tension" between two points ... (A) can be measured by the Rogowski method. See e.g. R. W. Pohl [3], pp. 76-80. For practical reasons this measurement is carried out as measurement of the impulse of an open voltage ( $\int Vdt$ ). When we imagine a Rogowski coil as having a negligible resistance and short-circuited with a resistance-free ammeter, then in this manner the "magnetic tension" can also be measured directly as a number of short-circuit ampere-turns.

Homogeneous field distribution can be obtained by using extension coils (see fig. 3.)

Table II. Comparative table of formulea

	OTHER T	and compared to the contract of the contract o		
	Rationalized Giorgi	$G = 3.10^{10}  \mathrm{cm/sec}$	Electrostatic c.g.s. $(c \approx 3.10^{10} \text{ cm/sec})$	Electromagnetic c.g.s. $(c \approx 3.10^{10}  \mathrm{cm/sec})$
Maxwell's laws.	$= - \overrightarrow{B} \dots$ $= \overrightarrow{D} + S \dots$ $= - \overrightarrow{e} \dots$	$= \dot{\mathbf{b}}/c$ $= \dot{\mathbf{b}}/c$ $= +4\pi$ $= +7$ $= -7$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	$D=\varepsilon E \ldots (\mathrm{C/m^2})$ $B=\mu H \ldots (\mathrm{Wb/m^2})$	$D = \varepsilon_r E$ $B = \mu_r H$	$D == c_r E$ $B = \mu_r H/c^2$	$D = c_r L/c_r$ $B = \mu_r H$
Law of induction	$V = -\dot{\phi}$ (V)	$V=-\dot{\phi}/c$	/	$V=-\dot{\phi}$
Plane capacitor	$C = \varepsilon A/s$ (F)	$C = \varepsilon_r A/4 \pi s \qquad \dots \text{ (cm)}$	$C = \varepsilon_r A/4 \pi s \dots (cm)$	
Sphere	$C = \varepsilon \cdot 4 \pi r \qquad \dots \text{ (F)}$	C = r (cm)	C = r (cm)	
Long coil (n turns)	$L = n^2  \mu  A/s  \dots \text{ (H)}$	$L = 4 \pi n^2 \mu_r A/s \dots \text{ (cm)}$		$L = 4 \pi n^2 \mu_r A/s \dots (cm)$
Field strength in long coil (n turns)	H = n I/s $(A/m)$	$H = 4 \pi n I/c s$ (oersted)		$H=4 \pi n I/s \dots (\text{oersted})$
Space density of energy	$\frac{1}{2}ED+\frac{1}{2}HB$ $(W\cdot sec/m^3)$	$\frac{1}{2} E D/4 \pi + \frac{1}{2} H B/4 \pi$ (erg/cm <sup>3</sup> )		
Force on a charge	K = QE (N)	K = QE (dyne)	K = Q E (dyne)	
Coulomb's law	$K = Q_1 Q_2/\varepsilon \cdot 4\pi r^2 \dots (N)$	$K = Q_1 Q_2/\epsilon_r r^2$ (dyne)	$K = Q_1 Q_2/\varepsilon_r r^2 \dots (\mathrm{dyne})$	
Force on charge in motion	$K = Q v B \qquad \dots (N)$	K = Q v B/c (dyne)		$K = Q v B \dots (dyne)$
Force on straight wire	K = IlB (N)	K = I I B/c (dyne)		$K = I l B \dots (dyne)$
Force between two parallel wires	$K = \mu I_1 I_2 l / 2 \pi r \dots (N)$	$K = 2\mu_r I_1 I_2 l/c^2 r \dots \text{ (dyne)}$		$K=2\mu_r I_1 I_2 l/r \ldots  ext{(dyne)}$
Force between to charged plates	$K = \frac{1}{2} Q E = \frac{1}{2} \Psi E \dots (N)$	$K=rac{1}{2}$ Q $E=rac{1}{2}$ A D $E/4$ $\pi$ (dyne)	$K = \frac{1}{2} Q E = \frac{1}{2} A D E/4 \pi$ (dyne)	
Force between to plane magnet poles	$K = \frac{1}{2} \Phi H \dots (N)$	$K = \frac{1}{2} \Phi H/4 \pi$ (dyne)		$K=rac{1}{2}\Phi H/4\pi\ldots({ m dyne})$

Table III. Comparison of the units of different systems

Notes: 1) It is to be borne in mind that the formulae denoting the relation between the various quantities may differ for the different systems (cf. table II).

<sup>2)</sup> In this table c is the numerical value of the velocity of light in m/sec: 2.99776  $\cdot$   $10^8 \approx 3 \cdot 10^8$ .

	Giorgi	Electrostatic c.g.s.	Electromagnetic c.g.s.	Gauss
	A	$1 \text{ e.s.u.} = 1/10c \approx 3.33 \cdot 10^{-10}$ (A)	$1 \text{ e.m.u.} = 10 \dots (A)$	$1 \text{ e.s.u.} = 1/10c \approx 3.33 \cdot 10^{-10} \dots (A)$
	Δ	1 e.s.u. = $c/10^6$ $\approx 300$ (V)	1 e.m.u. = $10^{-8}$ (V)	1 e.s.u. = $c/10^6 \approx 300$ (V)
	1 C  (coulomb) = 1  A·sec	$1 \text{ e.s.u.} = 1/10c \approx 3.33 \cdot 10^{-10}$ (C)	1 e.m.u. = 10 (C)	1 e.s.u. = $1/10c \approx 3.33 \cdot 10^{-10}$ (C)
	$1 \Omega = 1 \text{ V/A}$	1 e.s.u. = $c^2/10^5$ $\approx 9.10^{11}$ ( $\Omega$ )	1 e.m.u. = $10^{-9}$ ( $\Omega$ )	1 e.s.u. $= c^2/10^5 \approx 9.10^{11}$ $(\Omega)$
Capacitance C	$1  \mathrm{F} = 1  \mathrm{A·sec/V}$	$1 \text{ cm} = 10^5/c^2 \approx 1.11 \cdot 10^{-12}$ (F)	$\frac{1}{1}$ l e.m.u. = $10^9$ (F)	$1  ext{ cm} = 10^5/c^2 \approx 1.11 \cdot 10^{-12} \ \dots  ext{(F)}$
Self-induction L	1 H = 1 V·sec/A	1 e.s.u. = $c^2/10^5$ $\approx 9.10^{11}$ (H)	$1 \text{ cm} = 10^{-9} \dots (H)$	$1 \text{ cm} = 10^{-9} \dots (H)$
Electric flux $\Psi$ $(\equiv \iint D_n  \mathrm{d}A)$	$C = C$ $(\Psi = Q)$	1 e.s.u. = $10^{-1}/4\pi c \approx 2.65 \cdot 10^{-11}$ (C)	<sup>-11</sup> 1 e.m.u. = $10/4\pi \approx 7.96 \cdot 10^{-1}$ (C)	1 e.s.u. = $10^{-1}/4\pi c \approx 2.65 \cdot 10^{-11}$ (C)
placement, electric induction D	C/m <sup>2</sup>	1 e.s.u. = $10^3/4\pi c \approx 2.65 \cdot 10^{-7}$ (C/m <sup>2</sup> )	1 e.m.u. = $10^5/4\pi \approx 7.96 \cdot 10^3$ (C/m <sup>2</sup> )	1 e.s.u. = $10^3/4\pi c \approx 2.65 \cdot 10^{-7}$ (C/m <sup>2</sup> )
Electric field strength E	ν	1 e.s.u. = $c/10^4$ $\approx 3.10^4$ $(V/m)$	1 e.m.u. = $10^{-6}$ $(V/m)$	1 e.s.u. = $c/10^4 \approx 3.10^4$ $(V/m)$
Magnetic flux $\Phi$ ( $\equiv \lceil \lceil B_n  dA \rceil$ )	$\begin{array}{c} 1 \text{ Wb (weber)} \\ = 1 \text{ V·sec} \end{array}$	1 e.s.u. = $c/10^6$ $\approx 300$ (Wb)	1 maxwell = 10 <sup>-8</sup> (Wb)	$1 \text{ maxwell} = 10^{-8} \dots \text{ (Wb)}$
Magnetic induction B	$ m Wb/m^2$	$1 \text{ e.s.u.} = c/10^2 \approx 3.10^6 \dots (\text{Wb/m}^2)$	l gauss = $10^{-4}$ $({\rm Wb/m^2})$	1 gauss = $10^{-4}$ $({\rm Wb/m^2})$
Magnetic field strength H	A/m ·	$1 \text{ e.s.u.} = 10/4\pi c \approx 2.65 \cdot 10^{-9}$ (A/m)	1 oersted = $10^3/4\pi \approx 79.6$ (A/m)	1 oersted = $10^3/4\pi \approx 79.6$ $\dots$ (A/m)
	$\begin{array}{c} 1 \text{ N (newton)} \\ = 1 \text{ kg·m/sec}^2 \end{array}$	$1~\mathrm{dyne} = 1~\mathrm{g\cdot cm/sec^2} = 10^{-5}~\mathrm{N}$	$   1 \text{ dyne} = 1 \text{ g·cm/sec}^2 = 10^{-5} \text{ N}   1 \text{ dyne} $	1 dyne = $1 \text{ g·cm/sec}^2 = 10^{-5} \text{ N}$
	$\begin{array}{l} 1 \text{ N} \cdot \mathbf{m} = 1 \text{ W} \cdot \text{sec} \\ = 1 \text{ V} \cdot \mathbf{A} \cdot \text{sec} \\ = 1 \text{ J (joule)} \end{array}$	$egin{array}{ll} 1~\mathrm{erg} &=1~\mathrm{dyne\cdot cm} \ &=10^{-7}~\mathrm{N\cdot m} \end{array}$	$1 \text{ erg} = 1 \text{ dyne·cm}$ $= 10^{-7} \text{ N·m}$	$1 \text{ erg} = 1 \text{ dyne·cm}$ $= 10^{-7} \text{ N·m}$

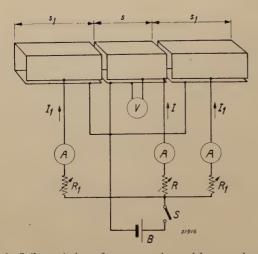


Fig. 3. Coil consisting of a centre piece with extension coils. Inhomogeneity of the field occurs at the ends. Only the voltage impulse of the centre piece is measured, where the field is homogeneous and equals I/s. A denotes ammeter; B battery, R and  $R_1$  variable resistors for making  $I_1/s_1 = I/s$ ; S switch for switching the currents I and  $I_1$  on and off; V ballistic voltmeter not consuming any current.

absolute volt and ampere has the value:

$$\mu_0 = 4\pi/10^7 \dots (H/m).$$

Now that we have learnt the meaning of  $\mu_0$  we can remember the definition of the absolute volt and ampere from the following: the product of V and A is given by the energy equation 1 N·m = 1 V·A·sec: the quotient of V and A is given by  $\mu_0 = 4\pi/10^7\dots (V\cdot \sec/A\cdot m).$  The reader will now understand why in this article these two formulae — upon which the whole Giorgi system can be built up — have been given special prominence by framing them.

It is common practice to indicate the permeability of a material magnetic medium as the product of its "relative permeability"  $\mu_r$  and  $\mu_0$ :

$$\mu = \mu_r \mu_0 \dots (H/m).$$

 $\mu_r$  is a non-dimensional number, the material constant known of old.

The force exercised upon a straight conductor of the length  $l \dots (m)$ , when a current  $I \dots (A)$  is

flowing through it and it is directed perpendicular to the direction of a homogeneous magnetic field, is at right-angles to these two directions and has the value:

$$K = IlB \dots (N).$$

Whenever it is required to regard the magnetic induction B as a force vector this can also be measured not in  $Wb/m^2$  but in the identical unit  $N/A \cdot m$ .

By means of this equation one can find the force between two parallel conductors by calculating the induction caused at  $I_2$  by the field strength due to  $I_1$ . It is with the aid of this force that the "Comité international des Poids et Mesures" has defined the absolute ampere  $^{10}$ ).

The foregoing should have made it clear how, by using the rationalized Giorgi system, one can deal in like manner with the important concepts of electromagnetism, viz. current field, electric field, magnetic field, and the phenomena related thereto.

For those who are accustomed to working with the older systems and now wish to change over to the rationalized Giorgi system it may well be useful to be able to compare the formulae and units of the old systems with those of the new one. For this purpose we have compiled tables II and III. For practical application particular attention is drawn to the note 1) to table III.

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<sup>10)</sup> W. de Groot [4], p. 58.

### THE EFFECT OF THE MELTING POINT AND THE VOLUME MAGNETO-STRICTION ON THE THERMAL EXPANSION OF ALLOYS

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When metal has to be sealed to glass it is first of all necessary that the expansion coefficient of the metal in a certain temperature range does not differ too much from that of the glass. It is investigated how the expansion coefficient of alloys is related to the drop in their melting point and what effect the volume magneto-striction has upon that coefficient in the case of alloys of ferromagnetic materials. An empirical rule is given for the composition of alloys with a small expansion coefficient. As a practical example the composition of an alloy consisting of iron, nickel, cobalt and copper which is suitable for sealing to hard glass is discussed.

Glass and metal can only be sealed together when both materials satisfy certain requirements. At temperatures lying between that for normal use and the minimum temperature at which one of these two materials is able to neutralize by plastic changes any stresses arising during the sealing of the glass to the metal, the expansion of the glass must within sufficiently narrow limits be equal to that of the metal <sup>1</sup>). In the second place the adhesion between the metal and the glass, either direct or via a skin of oxide, is of the greatest importance. Moreover, the materials have to answer a number of technological requirements, e.g. malleability, soldability, non-corroding, etc.

In this article we shall confine our discussions to those aspects of importance in the development of alloys which must satisfy certain requirements regarding thermal expansion. It will be shown that the thermal expansion coefficient of an alloy is related to its melting point and its magnetic properties. These relations make it possible to find from the data regarding the melting points on the one hand and the magnetic properties on the other hand an indication of the composition of alloys having expansion coefficients answering the requirements.

#### Expansion and melting point of elements

An empirical rule has long been known which gives the relation between the melting point and the linear thermal expansion for elements. According to this simple rule the total expansion from the absolute zero point to the melting point is the same for all elements. This is important, because usually the melting point is one of the best known data for any solid substance. The rule is in agreement with the fact that the higher the melting point the

greater are the binding forces of the lattice (and thus the greater the hardness) at a certain temperature, which means that the expansion is less. By way of illustration, in *table I* some data are given for three metals with greatly different melting points.

Table I. The melting point, the total linear expansion from the absolute zero point to the melting point, the expansion coefficient between 0 and 100  $^{\circ}$ C and the hardness of three metals.

Metal	Melting point	Total linear expansion	Expansion between 0° and 100 °C per °C	Vickers hardness
W Cu Al	3400 °C 1083 °C 660 °C	$ \begin{array}{c} 24 \cdot 10^{-3} \\ 23 \cdot 10^{-3} \\ 22 \cdot 10^{-3} \end{array} $	$43 \cdot 10^{-7} \\ 165 \cdot 10^{-7} \\ 240 \cdot 10^{-7}$	250 kg/mm <sup>2</sup> 35 kg/mm <sup>2</sup> 15 kg/mm <sup>2</sup>

From a list of 38 elements (metals and nonmetals) for which sufficient data on the expansion up to the melting point are known, we find for the total linear expansion

$$22.0\times10^{-3}+3.5\times10^{-3}$$
.

This relates to pure elements. It is exceptional, however, for an element to answer exactly the requirements in regard to thermal expansion. Yet there are cases where this is so. For instance tungsten and molybdenum can be used for sealing to hard glass, and platinum for sealing to soft glass. If, however, there are objections against using these elements, for instance because they are difficult to process (W and Mo) or because the material is too expensive (Pt), we have to investigate what allovs can be used for the purpose in view. Our thoughts then turn in the first place to real (homogeneous) alloys and not to a composition of different materials such as the so-called copper-clad wire, consisting of a nickel-iron core with a low expansion coefficient clad in copper with a high one

<sup>1)</sup> A. A. Padmos and J. de Vries, Stresses in glass and their measurement, Philips Techn. Rev. 9, 277-285, 1947 (No. 9).

In this last case a material can be obtained which has a well-matched average expansion in the radial direction but its use is restricted, by the inevitable anisotropy of the expansion, to special applications such as lead-in wires for lamps.

The alloys must be of such a composition that no phase changes take place within the temperature range in which they are to be used. Changes are almost always taking place in volume at the phase change and, moreover, there are usually great differences in the expansion coefficients between two phases of the same material .A familiar example of this latter phenomenon is the difference in the expansion coefficients  $\beta^2$  of the  $\alpha$ -phase and the  $\gamma$ -phase  $\beta^2$  of iron, the former being  $\beta^2$  and the latter  $\beta^2$  of iron. By adding suitable percentages of manganese or nickel the  $\gamma$ -phase of iron can be made stable at room-temperature. In that case the expansion coefficient of  $170 \cdot 10^{-7}$  just mentioned still holds.

The above-mentioned rule, in so far as it is used to indicate the change of the expansion due to an admixture, no longer applies for alloys. Nevertheless, from the melting point a conclusion can be drawn as to the extent of the thermal expansion.

### Melting point drop and thermal expansion of homogeneous alloys

The melting point of a metal is lowered when a second metal is added. It appears that in the case of homogeneous alloys a relation exists between the extent of this drop in melting point (when adding an alloy element) and the corresponding change in the expansion coefficient.

To explain this we shall start with iron. This metal is used preferably as the main component of an alloy for sealing not only because it is inexpensive but also because its expansion coefficient

<sup>2</sup>) In this article  $\beta$  is used to denote the linear expansion coefficient, which as is known, is defined for a specific substance by the relation  $\beta=1/l\cdot dl/dT$ , where l is the length at the temperature T. Within small ranges of temperature this coefficient is so constant that we may write  $l_{T'}=l_T\left[1+\beta\left(T'-T\right)\right]$ , where  $l_{T'}$  and  $l_T$  are the lengths of a bar at T' and T °C respectively.

 $(\beta=120\cdot 10^{-7})$  does not differ so very much from that of normal soft <sup>4</sup>) glass ( $\beta=95\cdot 10^{-7}$ ), so that the addition of a second metal need only cause a relatively small reduction of the expansion coefficient.

In table II the change of the linear expansion coefficient (between 0 and 400 °C) is compared with the change in melting point resulting from the addition of another metal to iron. It is to be noted that in both cases extrapolation has been carried out to very small percentages of the material added.

It appears that the less the added metal reduces the melting point of the iron, the lower is the expansion coefficient, whilst the greatest reduction of the melting point is even accompanied by a slight increases of the expansion coefficient.

Table II. The changes taking place in the melting point of iron  $\Delta T_s$  and in the linear expansion coefficient  $\Delta \beta$ , both per atomic percent, when a second element is added. In the last column the solubility of the added element is given.

Added element percent	$\Delta T_s$ per atomatic percent	$\Delta\beta$ between 0 and 400 °C per	Solubility in iron
Sn	— 10 °C	+ 0.31×10 <sup>-7</sup>	5 atom. %
Si	7	+ 0.04	25
Ni	<b>— 3.3</b> .	- 2.0	18
Co	1.3	0.9	75
$\mathbf{v}$	- 1.2	- 5.7	30
Al	0.5	- 0.9	20
Mo	0.5	- 6.7	4
Cr	0	5.9	40
W	0	8.3	3

The regularity of this phenomenon does not apply to quite the same extent in the case of cobalt and aluminium, for there the expansion coefficient is reduced less than would be expected from the general rule.

In fig. 1 the change in the expansion coefficient  $\Delta\beta$  of iron is plotted as a function of the quantity of material added. The initial slopes of the curves correspond to the values of  $\Delta\beta$  given in table II The diagram in fig. 1 also indicates the extent to which the expansion coefficient of soft glass differs from that of iron.

It appears that only three elements are suitable for combining with iron to form a metal which can be sealed. Only in the case of the binary alloys Fe-Cr, Fe-V and Fe-Co (with a high Co-content) is the expansion coefficient, as compared with that of pure iron, sufficiently reduced before the limit of solubility is reached; this is due not only to the

<sup>3)</sup> As is known, the iron atoms form a cubic lattice in both modifications ( $\alpha$  and  $\gamma$ ), but with this difference that in the case of alpha iron in addition to the corners the spaces in the middle of the cubic cells are also occupied (bodycentred cubic structure), whereas in the case of  $\gamma$ -iron in addition to the corners the spaces in the middle of the cube planes are occupied (face-centred cubic structure). The reason why the modification that is stable from 900 up to about 1400 °C is denoted by  $\gamma$  and not by  $\beta$  is becauses at about 700 °C (the Curie point of iron) the magnetic  $\alpha$ -iron stable at room temperature becomes nonmagnetic, and the non-magnetic phase having the same crystal structure as the magnetic phase is denoted by  $\beta$ -iron. The conversion at 900 °C might actually be called the  $\beta \rightarrow \gamma$  transition.

<sup>4)</sup> Soft glass has a high expansion coefficient ( $\beta=90$  to  $100\cdot 10^{-7}$ ); hard glass has a low expansion coefficient ( $\beta=40$  to  $50\cdot 16^{-7}$ ).

relatively high  $\Delta\beta$ /atomic percentage ratio but also to the satisfactory solubility of the added element in iron. Of the alloys mentioned here the chromium iron alloys are well known and employed as sealing metals.

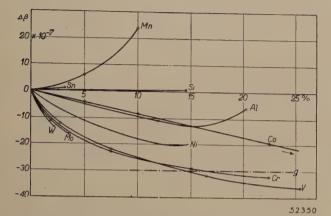


Fig. 1. Variation of the expansion coefficient of iron (between 0 and 400  $^{\circ}\mathrm{C}$ ) when a certain percentage of another metal is added in solution. The quantity of the second element is set out on a horizontal axis as an atomic percentage. The graph for Fe-Co, which runs almost straight, has to be extended further owing to the great solubility of this combination. The Fe-Al alloys show an irregularity due to a rearrangement in the lattice. The Fe-Mn alloys show differences in behaviour due to stabilisation of the gamma phase of iron at room temperature resulting from the addition of manganese. The horizontal dot-dash line denoted by g represents the expansion coefficient of soft glass compared with that of iron. It appears that only Fe-Cr, Fe-V and Fe-Co can be considered for sealing to soft glass.

With reference to fig. 1 it is also to be noted that alloys with an Si content greater than about 15 atomic percent have not been included. This is because with these alloys, just as the graph shows to be the case for Fe-A1, irregularities occur which are related to the fact that a rearrangement takes place in the lattice. By this it is to be understood that the A1 and Fe ions, or the Si and Fe ions, are no longer arbitrarily distributed among the available lattice points, but alternate one with the other in a certain order. This tends to effect the expansion, whilst moreover the change from order to disorder or vice versa may be accompanied by changes in volume.

We have included in fig. 1 also the Fe-Mn alloys previously mentioned to show by an example how the expansion coefficient of these alloys increases as a result of the stabilization of the  $\gamma$ -phase of iron at room temperature through the addition of manganese, in spite of the fact that the melting point of iron is scarcely affected by Mn. The behaviour of manganese therefore is not in agreement with the rule for the relation between  $\Delta T_s$  and  $\Delta \beta$  as represented in table II.

Ordered arrangement in the lattice and phase changes undoubtedly affect the expansion, but since both these effects occur in relatively small temperature ranges they are not of great importance, and sometimes even undesired, for the application of alloys as sealing metals. It is a different matter, however, with the magnetic influence upon the volume, to which we shall now give attention.

#### Ferromagnetic properties and thermal expansion

Under the influence of an external magnetic field a ferromagnetic material appears to undergo changes in its form. This phenomenon is called magnetostriction 5). A change takes place in dimensions in the direction of the magnetisation, with an opposite change in the directions perpendicular thereto. so that to a first approximation the volume is not changed. The change in the dimensions in the direction of magnetisation may be a lengthening or a shortening, the magneto-striction in the first case being said to be positive and in the second case negative. This change in length, which together with the magnetisation, reaches a saturation value, may be fairly considerable, amounting for instance to  $60 \cdot 10^{-6}$  of the length. Further, accurate measurements have shown that there is another effect. a change in volume of the ferro-magneticum, called volume magneto-striction, which, however, only reaches a perceptible value when the field strengths are very high. This volume magnetostriction may result in either an increase or a reduction of the volume.

In order to indicate the relations for these phenomena and the effect they may have upon thermal expansion, we have to consider more closely our ideas of a ferro-magnetic material. Ferromagnetism arises from the fact that, owing to certain exchange forces, adjacent lattice elements each possessing a magnetic moment tend to equalize these moments. Such a representation would lead one to expect that without an external field any ferromagneticum would always show the saturation magnetisation corresponding to the best possible parallel placing of all these primary minute magnets, under the influence on the one hand of the exchange forces and on the other hand of the forces disturbing this regularity, viz. the thermal motion. The fact that such a material nevertheless often shows outwardly only small magnetisation or even no ferromagnetism at all is explained by the circumstance that although the saturation magnetisation exists in small regions (Weiss's domains) the ferro-magneticum is built up from a large number of these small regions which have mutually different directions of the magnetic moment. By applying an

<sup>5)</sup> Striction = contraction.

external magnetic field, in the first place the magnetic moments af all individual regions may, as far as is possible, become orientated in parallel directions. In the second place, by applying very high external fields one can improve the parallelization of the separate moments within each region in so far as they are disturbed by thermal agitation. The effect first mentioned produces the normal saturation of the ferro-magneticum as determined by measurement in an external magnetic field of moderate strength, and with this parallel orientation of the magnetic moments of Weiss's domains linear magnetostriction occurs. The second effect, which increases somewhat the saturation of the ferro-magneticum, is accompanied by the volume magneto-striction mentioned above. From this we derive the important rule that the volume is a function of the saturation.

From the foregoing it will be clear that saturation may be influenced by very strong external magnetic fields as well as by temperature. Therefore, volume magneto-striction may arise both from external magnetic fields and from variations in temperature. This change in volume with temperature has to be superimposed upon the normal thermal expansion. Obviously this applies only for temperatures below the Curie point, because above that ferromagnetism disappears entirely.

In general volume magneto-striction and thus also its influence upon expansion is very small; only in exceptional cases is there any great effect. For pure iron  $1/V \cdot \mathrm{d}V/\mathrm{d}H$  (where V= the volume and H= the magnetic field) is about  $6 \cdot 10^{-10}$  per oersted  $^6$ ) and for pure nickel even 6 times as small.  $\mathrm{d}V/\mathrm{d}I$  (I= the magnetisation) can be calculated by multiplying the measured value of  $\mathrm{d}V/\mathrm{d}H$  by  $\mathrm{d}H/\mathrm{d}I$ . In the case of strong magnetic fields, where a linear relation exists between I and I, this is a constant factor for any particular material. Further, it is possible to deduce  $\mathrm{d}I/\mathrm{d}T$  from the curve giving the relation between the saturation I and the temperature I. The quantity that is of importance to us,  $\mathrm{d}V/\mathrm{d}I=\mathrm{d}V/\mathrm{d}I\cdot\mathrm{d}I/\mathrm{d}I$ , is then also known.

In the developing of alloys suitable for sealing to hard glass it is usually a matter of finding a composition having a lower expansion coefficient than that of the basic materials. This is attained when  $\mathrm{d}V/\mathrm{d}H$  and  $\mathrm{d}I/\mathrm{d}T$  are both large (in absolute value). (It is assumed that  $\mathrm{d}V/\mathrm{d}H$  is positive, because  $\mathrm{d}I/\mathrm{d}T$  is always negative.) It is therefore necessary to find alloy which satisfies these two conditions.

This can be explained further with reference to the diagram in fig. 2 relating to the iron-nickel system. It appears that an alloy can be formed in which the volume mangeto-striction reaches the exceptionally high value of  $3\cdot 10^{-8}$  per oersted. If, now, at the same time the value of  $\mathrm{d}I/\mathrm{d}T$  for this alloy is large, the expansion coefficient will be of the desired small value. In order to ascertain at what temperature a high value can be expected for this differential quotient we have to consider the variation of I as a function of temperature. It appears that I has a maximum value at the absolute zero point, and decreases from that point onwards, the decrease being rapid for temperatures not far

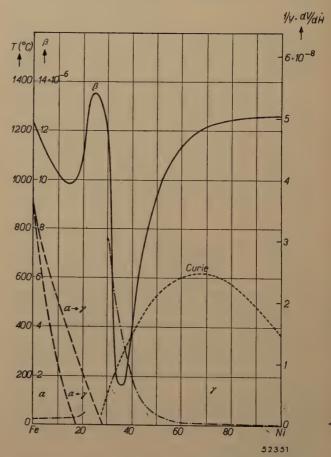


Fig. 2. Diagram of the Fe-Ni-system with the limits of the a and  $\gamma$ -phases, the value of the volume magnetostriction, the Curie points and the value of the linear expansions coefficient (between 0 and 100 °C). The alloy having a minimum expansion coefficient belongs to the gamma phase in a region which for the temperatures to be considered lies close to the  $a \rightarrow \gamma$  boundary. This alloy has a relatively low Curie point. On the left-hand axis we have the temperature and the value of the expansion coefficient (between 0 and 100 °C) and on the right the value of  $1/V \cdot dV/sH$  per oersted as a measure for the volume magneto-striction. The line denoted by  $\beta$ relates to the expansion coefficient and the dot-dash line denotes the volume magneto-striction; the latter is interrupted in the region where this value cannot be determined. curve marked Curie indicates how high the Curie temperature is for the alloys to be considered. For the phase transitions  $a \rightarrow \gamma$  and  $\gamma \rightarrow \alpha$  different lines have been drawn because it makes a difference in the results whether these transitions are brought about by heating or by cooling.

<sup>6)</sup> In Giorgi units  $1/V \cdot dV/dH = 48 \cdot 10^{-9}$  m/A.

below the Curie temperature  $T_1$ , where the zero value is reached. When an alloy is sought with a small expansion coefficient calculated for a range from 0 to  $T_2$ , the conditions in respect to  $\mathrm{d}I/\mathrm{d}T$  will only be met when  $T_2$  is not much lower than  $T_1$ ?). In practice what this amounts to is that the Curie temperature of the alloy required must lie round about the transformation point of glass.

Fig. 2 also gives for the iron-nickel system the Curie temperature of the alloys as a function of their composition. It is seen that the alloy with a very large volume magnetostriction has a sufficiently low Curie temperature and is therefore suitable for the purpose. This is confirmed by the graph for the expansion coefficient (between 0 and 100 °C) as a function of the composition of the alloy, plotted in the same diagram.

An examination of fig. 2 shows that a composition of the alloy with a minimum expansion coefficient is not exactly the same as that of the alloy with a maximum volume magnetostriction. In this connection it has to be considered that the expansion coefficient is given for the range 0-100 °C and the volume magneto-striction has been measured at a temperature of 4 °C. The graph for the volume magneto-striction is interrupted at the point where the value of this quantity cannot be determined.

From fig. 2 we see that the alloy consisting of 63% Fe and 37% Ni has a very low expansion coefficient ( $\beta=1.5\times10^{-6}$ ). This alloy is of practically the same composition as the alloy "Invar", which has long been known and is used for making measuring rods, clock pendulums and suchlike.

Further it is to be seen from the diagram that in the iron-nickel system the alloy with a large volume magneto-striction belongs to the γ-phase in a region which, for the temperatures to be considered, lies close to the  $a \rightarrow \gamma$  limit. We have found confirmation of this emperical rule in the examination of other systems too. When we have to do with unknown systems this rule therefore offers an indication as to what alloys should be considered for examination. This also explains why we have indicated the phase limits in the graphs showing the variation of the volume magneto-striction and the expansion coefficient as functions of the composition. The transitions from the alpha to the gamma phase and from the gamma to the alpha phase are represented by separate lines, because different results are obtained with increasing and decreasing temperatures.

It is obvious that, as found with the Fe-Ni system, with other systems a large volume magneto-striction alone is not sufficient either; in addition,  $\mathrm{d}I/\mathrm{d}T$  must be sufficiently large and  $\mathrm{d}V/\mathrm{d}H$  must have the desired sign.

Theoretically it is feasible that by a suitable choice of components an alloy can be made which has a negative expansion coefficient. Fig. 3

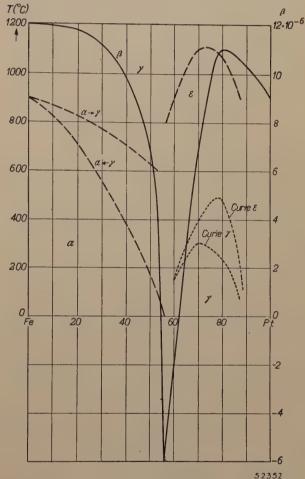


Fig. 3. The Fe-Pt system with the phase limits at room temperature, the Curie point value (both for the  $\gamma$  and for the  $\varepsilon$ -phase) and the value of the linear expansion coefficient (between 0 and  $100~^\circ\text{C}$ ).  $\varepsilon$ =a body centred cubic phase in this system differing from the  $\alpha$ -phase and partly bounded by the broken line. On the left-hand side we have the temperature and on the right the value of the expansion coefficient. It appears that the composition of the alloy can be chosen in such a way as to give a negative expansion coefficient. Here again we find confirmation that the alloy having a minimum expansion coefficient belongs to the  $\gamma$ -phase in a region which, for the temperatures to be considered, is close to the  $\alpha \to \gamma$  boundary.

shows that in the iron-platinum system this can indeed be done. Here again we find confirmation that the alloy with the minimum expansion coefficient belongs to the  $\gamma$ -phase and has a composition which, in the diagram of the system, lies close to the limit between the alpha and the gamma phases (at room temperature), whilst for this composition the Curie temperature is fairly low.

<sup>7)</sup> This is confirmed by the graph of σ (the saturation magnetisation per gram) for a Fe-Ni-Co alloy in fig. 7 in the continuation to this article.

In the case of ternary systems also, we have found confirmation of the empirical rule mentioned above. Of course it also holds here that a large volume magneto-striction is not sufficient to give an alloy a low expansion coefficient, but that  $\mathrm{d}I/\mathrm{d}V$  must be sufficiently large as well.

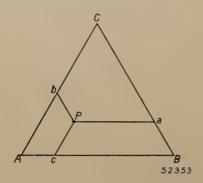


Fig. 4. In the equilateral triangle ABC, Pa + Pb + Pc = AB.

It is obvious, for these systems also, that the necessary curves for the volume magneto-striction, the Currie temperature and the expansion coefficient should be drawn in one diagram together with the phase limits. It is to be noted that when plotting such a diagram for a ternary system one usually starts from an equilateral triangle. The corners then correspond to the pure components A, B and C of the alloy, with the sides of the triangle representing the three binary systems AB, BC and CA, whilst a point inside the triangle characterizes a certain ternary alloy. Here we employ the theorems of planemetry which says that in an equilateral triangle the sum of the lines drawn from a point inside the triangle parallel to the three sides equals the side of the triangle. Thus in fig, 4Pa + Pb + Pc = AB. This enables us

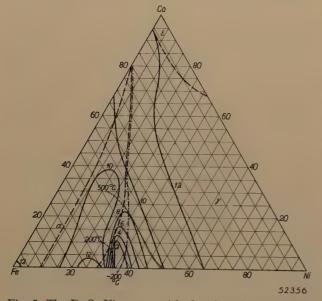


Fig. 5. The Fe-Co-Ni system with the phase limits at room temperature (——— lines) and, as far as the  $\gamma-\sigma$  transition is concerned, at  $-200\,^{\circ}\mathrm{C}$  (dot-dash lines). Further it is indicated what compositions have Curie point temperatures of 200 and 500  $^{\circ}\mathrm{C}$  (----- lines), whilst a number of lines are given for equal expansion coefficients measured between 30 and 100  $^{\circ}\mathrm{C}$  (full lines). The numbering of the latter curves indicates the value of this expansion coefficient  $\times$  106.

to find a point iside the triangle corresponding to an alloy composed for instance of 45% of the element A, 30% of the element B and 25% of the element C.

In such a phase diagram it is possible to set out the properties of the alloys as functions of their composition. For instance we can plot the phase limits at a certain temperature or draw lines showing what compositions have a certain Curie temperature. Lines can also be drawn between the points corresponding to compositions all having the same expansion coefficient.

Fig. 5 is an example of a phase diagram for a ternary system, viz. for the Fe-Ni-Co system. Here the phase limits are given at room temperature. It also shows with what compositions the Curie temperature is respectively 200 and 500 °C and

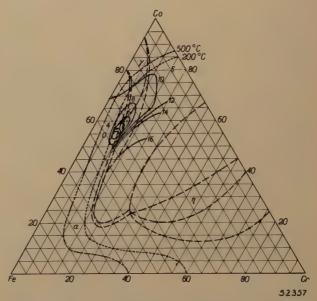


Fig. 6. The Fe-Co-Cr systems with phase limits at room temperature (——— lines), the composition of the alloys having a a Curie point temperature of 200 and 500 °C (---- lines) and a number of lines af equal expansion coefficient measured between 20 and 60 °C (full lines.) The numbering of the latter curves indicates the expansion coefficient  $\times$  106.

gives a number of lines of equal expansion coefficient. It appears that by a suitable choice of components alloys can be produced with zero expansion coefficient. These alloys belong to the gamma phase in a region close to the transsition to the alpha phase (at the temperatures to be considered) with a relatively low Curie temperature. In fig. 5 the limit between the alpha and the gamma phases at  $-200\,^{\circ}\mathrm{C}$  is also indicated because this will be referred to later on.

As a second example fig. 6 gives the phase diagram for the Fe-Cr-Co system. This shows the phase limits at room temperature, a number of lines of equal expansion coefficient (measured between 20 and 60  $^{\circ}$ C) and the compositions of the alloys having a Curie temperature of 200 and 500  $^{\circ}$ C respectively.

Here again we find confirmation of the rule that the alloys with small expansion coefficient lie in a region of the gamma phase close to the limit (for the working teperatures) of the alpha phase and at a point where the Curie temperature is sufficiently low.

#### Composing an alloy for sealing to hard glass

After this discussion of the two conditions which have to be met (high volume magneto-striction and relatively low Curie point) one may have obtained the impression that it is a simple matter to compose alloys for sealing glass to metal. In actual practice, however, difficulties are encountered. This is particularly the case when the metal has to be sealed to hard glass, i.e. glass with a high softening temperature and a small expansion coefficient. So far it has not even been found possible to find alloys suitable for sealing to glasses having an expansion coefficient less than about  $40 \cdot 10^{-7}$ .

In the first place it has to be borne in mind that the sealing is done close to the limit between the alpha and the gamma phases, with the risk that owing to the occurrence of the alpha phase, which has no considerable volume magneto-striction, the expansion coefficient again assumes the normal value of the  $\alpha$ -phase and thus becomes much too high.

In the second place, while the Curie point has to be relatively low, it must be high enough so as not to remain too far below the softening temperature of hard glass. The magnetic effect upon the expansion is of course only possible below the

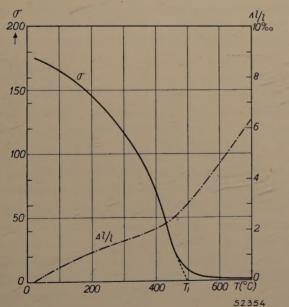


Fig. 7. Variation of magnetic saturation per gram ( $\sigma = I_s/d$ , where  $I_s$  is the normal saturation and d the density in grams) as a function of the temperature for a Fe-Ni-Co alloy having a small expansion coefficient below the Curie point temperature  $T_1$ . For the same alloy also the curve for the expansion  $\Delta l/l$  is plotted as a function of the temperature.

Curie point; above that even the very high expansion coefficient of the  $\gamma$ -phase arises.

We shall now deal with an example of the composition of a metal alloy for sealing to hard glass. In the past only molybdenum or tungsten could be used for this purpose on account of the expansion

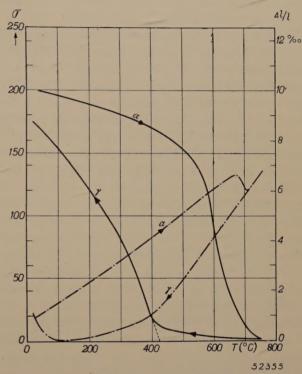


Fig. 8. Curves showing the expansion  $\Delta l/l$  (dot-dash lines) and the magnetic saturation per gram  $\sigma$  as function of the temperature (full lines) for a Fe-Ni-Co alloy which may occur at the same temperature not only in the gamma phase but also in the alpha phase. (The alpha phase is obtained by first strongly cooling the material down.) The temperature is first raised to 750 °C and then lowered to room temperature, the curves thereby following the direction indicated by arrows.

coefficient having to be so low. We shall now take, however, the Fe-Ni-Co system, the phase diagram for which is given in fig. 5. It will be found that with this system it is just possible to make a suitable alloy for sealing to hard glass. For grades of glass intermediate between hard glass and quartz glass it has not been possible to find a suitable alloy for two reasons: the expansion coefficients of these kinds of glass are still lower than that of hard glass (less than  $50 \cdot 10^{-7}$ ) and their softening points are too high, so that too high Curie temperatures are required for the sealing materials.

The conditions arising in our case may be seen from figs. 7 and 8, relating to two Fe-Ni-Co alloys, in the first of which the Ni content is somewhat higher and the Fe content somewhat lower than in the second alloy. In both diagrams the expansion and the saturation magnetisation are plotted as functions of the temperature. This

slope of the saturation-temperature curve shows that for temperatures not far below the Curie point the value of dI/dT is fairly high. From the slope of the expansion curve we can derive the value of the expansion coefficient  $\beta$ . Fig. 7 represents the position found with many such alloys. Above the Curie point the expansion coefficient of this material has the high value of  $155 \cdot 10^{-7}$ . At the Curie point the expansion coefficient begins to drop and from  $400\,^{\circ}$ C onwards  $\beta$  drops to the rather small value of  $60 \cdot 10^{-7}$ , which however is still too high for hard glass. In the second alloy (to which fig. 8 relates) the Ni content is reduced so as to reach a lower expansion coefficient below the Curie point. It is found that  $\beta$  is no more than  $25 \cdot 10^{-7}$ between 100 and 300 °C. But on the other hand there are the great disadvantages that owing to this reduction of the Ni content the Curie point is too low with respect to the softening point of the glass, and, as shown in fig. 5, at low temperatures the gamma phase is no longer stable. From this phase diagram it is to be deduced that at a tempeture of -200 °C the limit between the alpha and the gamma phases is partly shifted to the right. Consequently when this alloy is strongly cooled down, for instance in liquid hydrogen to -183 °C, the material is almost completely transformed into the alpha phase, with the result that the expansion coefficient ( $\beta = 86 \cdot 10^{-7}$ ) and also the saturation temperature curve correspond to those of a normal alpha phase. When heating to 700 °C this alpha phase changes into the gamma phase. This transition is accompanied by a change in volume 8), in this particular case a reduction, and at still higher temperatures the expansion coefficient has the normal value ( $\beta = 160 \cdot 10^{-7}$ ) of the gamma phase. Upon the temperature being lowered again, the gamma phase continues to exist with its high expansion coefficient until (at about 425 °C) the Curie temperature of this phase is reached and the expansion coefficient has dropped to the very low value of  $\beta=25\cdot 10^{-7}$ . At 100 °C however this gamma phase again begins to change into the alpha phase, with all the attendant objections of changes in volume and increasing expansion coefficient.

Although it has been found possible to produce in this manner an Fe-Ni-Co alloy suitable for sealing to hard glass, the transition from the gamma to to the alpha phase at low temperature remains an undesirable feature of this material. Since it is established that with ternary systems no improvement can be made in this respect, the question arises whether the desired change can be brought about by adding a fourth component. One might think of elements which promote gamma formation in iron, such as Mn, Cu, C znd N, but most of these elements are unsuitable. Owing to their low degree of solubility C and N have to be cut out in practice, whilst Mn reduces the Curie point far too much. Copper, however, can be used because when a small percentage of Ni is replaced by Cu not only is the temperature lowered at which the  $\gamma \rightarrow a$  transition takes place but at the same time the Curie point is slightly raised, both of which changes are favourable for fusing the alloy to hard glass. This is proved by table III. By comparing line 4 with line 1, 5 with 2 and 6 with 3 one finds that by adding this fourth element an alloy is obtained possessing properties which answer the requirements.

Table III. The expansion coefficient (for the range 0 to 350 °C) the Curie point and the temperature level of the  $\gamma \rightarrow a$  transition for some alloys of the Fe-Ni-Co-Cu system.

Ni	Co	Cu	Fe <sup>9</sup> )	β <sub>0-350 °C</sub>	Curie- point	$\begin{array}{c} \gamma \longrightarrow \alpha \text{-}\\ \text{trans-}\\ \text{ition} \end{array}$
26.7%	17.2%	_	56.1%	27 · 10-7	390 °C	+ 100 °C
28.9	17.3		53.8	55	430	- 30
29.6	17.5		52.9	61	460	183
26.5	17.5	1.0%	55.0	35	420	+ 20
26.5	17.5	2.0	54.0	48	450	- 80
26.5	17.5	3.0	53.0	59	480	— 183

<sup>9)</sup> Possibly with impurities.

<sup>&</sup>lt;sup>8</sup>) It is possible that the position of the Curie point may have some influence upon this change, because if it is so arranged that the  $a \rightarrow \gamma$  transition takes place not exactly at a temperature of the a Curie point then one finds separate irregularities for both effects.

### ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

Reprints of these papers not marked with an asterisk can be obtained free of charge upon application to the Administration of the Research Laboratory, Kastanjelaan, Eindhoven, Netherlands.

1758: D. Polder: Nature of the hydrogen bond in patassium hydrogen fluoride (Nature 160, 870, 1947, Dec. 20).

The doublet 1450, 1222 cm<sup>-1</sup> in the infrared absorption spectrum of KHF<sub>2</sub> is ascribed by Ketelaar to a doubling of the fundamental asymmetric frequency  $v_3$ , belonging to the vibration of a proton between two F-ions, due to a double minimum of the potential energy (analogeous to what happens in the NH<sub>3</sub> molecule). The validity of this conjecture is checked by measuring the temperature coefficient of the dielectric constant ( $\varepsilon^{-1}$  d $\varepsilon$ /dT). A value 2.10<sup>-4</sup> is found between T=80 °K and T=300 °K, whereas Ketelaar's data would yield a negative temperature coefficient between 0.01 and 0.001.

1759: C. J. Bouwkamp: A study of Bessel functions in connection with the problem of two mutually attracting circular discs (Proc. Kon. Ned. Akad. Wetenschappen, Amsterdam 50, 485-497, 1947, No. 9)

Calculation of the potential energy of two coplanar, circular, homogeneous discs for an arbitrary law of force, in terms of integrals containing Bessel functions.

Sepcial attention is paid to the case where the potential energy between two point masses varies with the distance r as  $r^{-n}$ . The integrals so obtained are evaluated in terms of elementary functions, complete elliptic integrals of the first and second kinds, and hypergeometric functions.

R 66: K. F. Niessen: Indication of landing courses independent of weather conditions I. (Philips Res. Rep. 3, 1-12, 1948, No. 1).

Discussion of the indication of straight landing courses with a small angle of elevation, independent of changes in the electric constants of the ground (as produced, for instance, by rain and snowfall). In this first part only infinitely small dipoles at unequal heights are considered.

R 67: A. van der Ziel and A. Versnel: Induced grid noise and total-emission noise (Philips Res. Rep. 3, 13-23, 1948, No. 1).

In space-charge-limited triodes the fluctuations in the number of electrons that have sufficient

energy to pass the potential minimum give rise, by electric induction, to a noise current flowing to the grid; the fluctuations in the number of the electrons returning in front of the potential minimum give rise, by electric induction, to a noise current flowing from cathode to grid. The first noise current is generally called "induced grid noise" and the latter "total-emission noise". Measurements are given of the noise resonance curve of the input circuit of pentodes at  $7 \cdot 25$  m wavelength. It is shown that the asymmetry of the noise resonance curve of the input circuit of pentodes at u.h.f. is due to the phase relation between the induced grid noise and the normal shot-effect noise.

Further, it is shown that for diodes in the cut-off region at 7.25 m wavelength the total-emission noise may be described by assuming that the equivalent noise temperature of the "total emission conductance" is equal to the cathode temperature.

R 68: B. D. H. Tellegen: The determination of the integration constants when calculating transient phenomena (Philips Res. Rep. 3, 24-36, 1948, No. 1).

A network is considered containing a voltage source v under the influence of which a current i flows in a certain branch. A method is given for calculating from the differential equation connecting i and v the discontinuities in i and its derivatives resulting from discontinuities in v and its derivatives. The method is applied to the calculation of periodic phenomena caused by periodic sources containing discontinuities.

R 69: H. J. Lindenhovius and J. C. van der Breggen: The measurement of permeability and magnetic losses of non-conducting ferromagnetic material at high frequencies (Philips Res. Rep. 3, 37-45, 1948, No. 1).

A method is described for a rapid and accurate determination of the permeability and the magnetic losses of non-conducting ferromagnetic material.

For frequencies between 30 and 300 Mc/s this method makes use of a coaxial cavity resonator with end-capacitance, accurately tuned to the frequency of an oscillator to which it is coupled. One measures the changes in the resonance frequency

of this cavity resonator after successively inserting, concentrically with the inner conductor, a ring made of the ferromagnetic material to be tested and a ring of the same size made of a well-conducting metal. The susceptibility ( $\chi=\mu-1$ ) of the ferromagnetic material equals the ratio of these two frequency changes.

The magnetic losses can easily be computed from the band widths of the cavity resonator with and without the ferromagnetic ring. For frequencies above 300 Mc/s a coaxial Lecher system is used instead of the cavity resonator. A minor complication of a dielectric nature then arising is eliminated in a simple way. The equations are the same as for the cavity resonator.

Some data obtained with compressed iron powder and with "ferroxcube" are given.

R 70: F. A. Kröger, J. M. Stevels and Th. P. J. Botden: The influence of hexavalent uranium in glass (Philips Res. Rep. 3, 46-48, 1948, No.1.)

In glasses hexavalent uranium may be present as uranyl groups or as uranate groups. The uranyl groups give rise to fluorescence at room temperature, but both uranyl and uranate groups show fluorescence at low temperatures. It is shown that there is a direct relation between the emission bands and the absorption bands.

R 71: W. J. Oosterkamp: The heat dissipation in the anode of an X-ray tube, I (Philips Res. Rep. 3, 49-59, 1948, No. 1).

The life of an X-ray tube is often determined by

the rate of evaporation of the target, and hence by the maximum temperature occurring during an exposure. For a given inflow of heat through the focus, the temperature of the target may be computed from the equations of heat conduction. The requisite general equations are developed and are then applied to the problem of loads of short duration in tubes with stationary anodes.

R 72: J. F. Klinkhamer: Emperical determination of wave-filter transfer functions with specified properties, I (Philips Res. Rep. 3, 60-80, 1948, nr. 1).

A method is decribed for determining wave-filter transfer functions  $z(\lambda) = e_u/e_i$  with specified properties by means of measurements in an electrolytic tank, eu being the output voltage, e; the input voltage, and  $\lambda$  the complex frequency parameter. At the same time other functions are discussed which are equally useful in calculating the filter-network performance (the "characteristic functions" of Piloty). The position of the transmission bands, the permissible variation of the attenuation within these bands, and the position and the minimum attenuation of the attenuation bands are supposed to be given. As the method is applicable to filters with several transmission and attenuation bands, not necessarily equal in attenuation qualities, the new method is more general than that of Cauer, though it bears a close relation to the latter. In the case of one transmission band and one attenuation band, and in the case of several transmission and attenuation bands with equal attenuation qualities, the results of the two methods are identical.